

THE STRUCTURE SEMIOTIC 'MYSTERY' OF THE GRAPHS

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Abstract: There is under investigation the structure of graphs. It is a method, where on the ground of 'semiotic invariants' be constructed a canonical presentation mode of the graphs, which can be used to solve some classical problems in a compact non-classical way, such as girths, cliques, parts, orbits, symmetry properties, regularities, isomorphism, reconstructions and others.

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Introduction

Structure semiotics [1], specify, *semiotics of the structure* is a research domain in the liminary of *graph theory* and *semiotics*. Under consideration is the *structure of a graph* as such. It constitutes a system of heuristic methods. On the aspect of *semiotics* it is touch with one of many object-orientated semiotics.

The arising of structure semiotic ‘mystery’ of the graphs is simple and reasoned. At one time to remain me in one’s memory a proposition of system theorists: ‘*Graph is an explicate of the structure*’. Indeed, analysis the classical definitions of structure concept take to the same. For differentiation a structure at another is the isomorphism testing too perfunctory, it no give information about the principal differences between structures. In operation of working out the canonical forms of the structure were arise the ‘semiotic invariants’. Their analysis shows, that to essential property of a structure is its symmetry attributes. Etc.

Structure (Latin word *structura* (inner)building) is a *philosophical category*, that is defined as *a steady relation or organizing of system elements* [2]. Organizing is an integral phenomenon at connectivity, branching, regularities, symmetry etc. On the other side, structure is an abstraction of the system, its ‘skeleton’, where its elements and their relationships be lose at their empirical meanings and to substrate is only its organizing. To note, that to adequate ‘language’ for express the structure of a cognizable object are *mathematical means*, concepts and methods. For the concept of structure is inherent an singular, at the same time an universal relation type – *composition* – that is expressable mathematically by equations, matrices, graphs etc. The *qualitative differences* between structural elements depend on their connectivity mode in structure – at their **positions** in the structure (example on graphite and diamond). And so, structure is a formation of mutually related elements that is presentable in **graph** form. To note, that the exact definition of structure give by **isomorphism** concept. Structure is the **canonical representation** of a *class the isomorphic graphs*. To originators of structure concept mistake for Immanuel Kant and Georg Wilhelm Friedrich Hegel'

Semiotics is a science of *signs* and *sign systems*. Deal with study the *meaning, communication- and interpretation processes*. A **sign** in semiotic system is an entity which signifies another entity. By Ch. Peirce is sign related with *cognition and thinking*. It includes the study of how *meaning* is constructed and understood. The ranges of uses of signs are varied. They might include: the indication or mark of something, a display of a message, a signal to draw attention, evidence of an underlying cause (for instance, the symptoms of a disease are signs of the disease), a character for a mathematical operation or representation, a body gesture, etc. To structural signs are specific **semiotic invariants** of a graph, what have intrinsic meaning.

Certainly differs structure semiotic vision of the graphs in some respects at conventional. Emerge new truths about graphs. It is no strange if an object is given on the various points. Truth is a belief (option) to a phenomenon that is cognized on a certain aspect. In general case is each phenomenon (system) many-aspect. Plurality the aspects no make the searching of own truth to needless. But none truth not may become to dogma. From graphs also can be makes out various truths. For example, well-known *four colour problem* is proved three times: by K. Appel, W. Haken in 1976 [3], Ashay Dhawadker in 2000 [4] and N. Robertson et al in 2003 year [5], everyone by his aspect or vision.

Structure semiotic vision not mystery, touch with an aspect of reality. Difficulty are only with getting rid of habits. For example, difficulty owns that *structure* is such a phenomenon what stays

invariable for isomorphic objects, and only these. Is it mystical? Be wont with verbal understandings, as institution structure, expression structure, rock structure etc.

What is a **graph**? On the structure semiotic aspect is the graph a formation of abstract elements and their adjacent pairs that has a *fixed structure* that is under investigation. Such aspect determines the approach to graphs.

A graph G is expressed by its *system of semiotic invariants* in the form of a *sign matrix* S . Sign matrix S represents the *structure* GS of a graph and constitute its *canonical form* with exactness up to isomorphism. If graphs G_A, G_B, G_C, \dots are *isomorphic* then corresponding sign matrices are *equivalent* $S_A \sim S_B \sim S_C \sim \dots$, i.e. the graphs have the same structure GS . The research of the structure takes place just on the ground of a semiotic sign matrix S .

A most essential structural attribute is *symmetry*. Also there are various visions. Deeply is rooted the understanding about symmetry as a kind of reflection.

Symmetry (A) (Greek word *symmetria*) is a property of an object, where the components that are placed at the same distance from a centre or axis are similar [2].

The ‘reflective’ concept is in mathematics defined as: a) a property of a figure to transform to itself, for example isometrically; b) property of binary relation $xRy \leftrightarrow yRx$; c) In graph theory is symmetry treated also on the aspect of directed graphs.

In fact is the symmetry concept more extend.

Symmetry (B) is a structural attribute that is expressed as a regular repetition (recurrence) of similar components (parts, particles) of an object in space and/or time [6].

The concept of symmetry **(B)** is *in mathematics* is defined as the existence the transitivity domain of automorphisms or *orbits* in $AutG$, i.e. in a graph G . An orbit is practically an *equivalence class*, whereof elements have the *equal positions* in structure GS . This conformity of positions be expressed as *symmetry* of structure. Essential are the relationships between *automorphisms, equal positions, orbits and semiotic invariants*. There exist symmetry by elements and symmetry by element pairs, the *symmetry kinds* are defined. Symmetry is *measurable*. Its value is maximum, if there exists only one orbit and its value is 0, if the number of orbits corresponds to the number of elements.

There exist *orbit structures* as partial structures, that are suitable means for recognition the various structural properties.

In the structure can be exist various *regularities*, such as *distance, girth, clique and strong regularity* that are mutually related. There are under investigation also the *bi-, tri-, quadro- and others cliques*.

Solution of the *reconstruction problem* takes place by *isomorphism classes* of greatest sub- and smallest super-graphs. On the structure semiotic aspect is the formulation of Ulam’s Conjecture too destructive.