2. SEMIOTIC MODEL AS TEXT OF STRUCTURE

For studying the structural properties is necessary *knowing the pair signs*, understand their meaning, ability *to read the semiotic model.*

2.1. Lexicology of semiotic model

Propositions 2.1. The structural ***meanings*** of ***pair signs ±d.n.q.***$_{ij}$:

P2.1.1. Pair sign in the form ***–d.n.q.***$_{ij}$***=*** *–∞.2.0* show a ***disconnected vertex pair.***
P2.1.2. Pair sign in the form ***–d.n.q.***$_{ij}$ show a ***simple path*** or their *assemblage* with length ***d*** and we call it ***path sign.***
For example: *–2.3.2* shows a 2-path; *–3.4.3* show a 3-path; *–7.8.7* show a 7-path, etc.
P2.1.3. Pair sign with greatest absolute value *max*|***–d***| show the ***diameter*** of the structure.
P2.1.4. Pair sign *+****dnq=+1.2.1*** show a ***link of a branch.***
P2.1.5. Pair sign in the form ***(+d=2).n.q*** show ***tiangularity,*** i.e. existence of 3-girths or -cliques.
P2.1.6. Pair sign +***dnq***, where +***d>2*** show a ***girth (cycle)*** or their *assemblage* with length ***d+1*** and it is called ***girth sign.***
Girth signs are, for example, ***+3.4.4*** for 4-girth; +***4.5.5*** for 5-girth; …+***7.8.8*** for 8-girth etc. If the length +***d*** no correspond with the numbers of vertices ***n*** and edges ***q***, then is touch with any mutually intercrossed *(d+1)-girths.*
P2.1.7. Pair sign in the form ***(+d=2).n.(q=n(n–1):2)*** show a ***clique*** and we call it ***clique sign.***
For example: ***+1.2.1*** show a 2-clique; ***+2.3.3*** – 3-clique; +***2.4.6*** – 4-clique; +***2.5.10*** – 5-clique; ***+2.6.15*** – 6-clique; …, +***2.13.78*** – 13-clique etc. Clique sign is a ***complete invariant*** of the clique, i.e. to clique sign correspond only a clique, to construct a "non-clique" is impossible.
P2.1.8. Pair sign in the form ***(+d=3).(n=a+b).(q=a×b)*** show a ***bi-clique*** with the number of elements ***a*** and ***b*** in the parts**.**

Semiotic model as a *text of structure* contain all the relevant information pro structure. Let's begin with the most simple – *regularities.*

Proposition 2.2. Read out the ***regularities:***

P2.2.1. Graph, where the numbers of pair(+)signs ***+dnq*** in all the rows ***i*** of semiotic model are equal is ***(degree)-regular.***
P2.2.2. Graph, where the partial signs ***–d*** of all the pair(–)signs ***–dnq*** in semiotic model are equal is ***distance-regular***.
On Example 2.1 showed Petersen graph with its pair(–)sign *–2.3.3* is *2-distance-regular.*
P2.2.3. Graph, where the partial signs ***+d*** of all the pair(+)signs ***+dnq*** in semiotic model are equal is ***girth-regular.***
In girth-regular graph belong all the ***n*** vertices the same number times to girth with length ***n–a***. For example, Petersen graph with its pair(+)sign ***+4.10.15*** is *5-girth-regular* (see Example 2.1).
P2.2.4. Graph, where the numbers of clique signs *+(d=2).n.(q=n(n–1):2)* in all the rows ***i*** of semiotic model ***S*** are equal is ***clique-regular.***
In clique-regular graph belong all the ***n*** vertices the same number times to clique with power ***n–b.*** If *a transitive* graph self not clique, then there cannot exist a single clique, it can be only clique regular. Unfortunately are almost all the clique algorithms oriented to recognition a *single clique.* For example, in complement of Petersen graph to fix usually a single *4-clique.* In fact is the complement *4-clique-regular.*

P2.2.5, A degree and 2-distance- regular graph, where in case of each pair(–)sign ***–2nq*** be found a pair(+)sign ***+2(n-a)(q-b)***, where for each ***n*** and ***q*** are ***a*** and ***b*** constant, is ***strongly regular.*** Graph said ***strongly regular*** with parameters (*k,**a,b***) if it is a *k-degree-regular* incomplete connected graph such that any two adjacent vertices have exactly ***a***≥0 common neighbors and any two non-adjacent vertices have ***b***≥1 common neighbors. For example, pair signs of a Weissfeiler's strongly regular graph are *–2.8.20, –2.8.19, –2.8.18* and *+2.7.13, +2.7.14, +2.7.15* (Example 3.1). Petersen graph is bisymmetric and *strongly regular.*

Ideology of all the algorithms of clique-finding started from the aim to isolate the vertices of maximum clique at others vertices. Such methods have been developed. Unfortunately, not have interest for the clique regularity.

Proposition 2.3. A *transitive,* i.e. vertex symmetric graph is ***girth-*** or/and ***clique-regular,*** and contrariwise.

Example 2.1. Pair signs, sign matrices and the lists of partial girths and cliques of Petersen-s graph ***PET*** and its complement ***PETC***:

```
               A:-2.3.2; B:+4.10.15.

|  1   2   3   4   5   6   7   8   9  10|   i  AB   k
   0   B  -A  -A   B   B  -A  -A  -A  -A|   1  63   1
       0   B  -A  -A  -A   B  -A  -A  -A|   2  63   1
           0   B  -A  -A  -A   B  -A  -A|   3  63   1
               0   B  -A  -A  -A   B  -A|   4  63   1
                   0  -A  -A  -A  -A   B|   5  63   1
                       0  -A   B   B  -A|   6  63   1
                           0  -A   B   B|   7  63   1
                               0  -A   B|   8  63   1
                                   0  -A|   9  63   1
                                       0|  10  63   1
```

```
A:-2.6.12; B:+2.5.8.

|  1   2   3   4   5   6   7   8   9  10|   AB
   0  -A   B   B  -A  -A   B   B   B   B|   36
       0  -A   B   B   B  -A   B   B   B|   36
           0  -A   B   B   B  -A   B   B|   36
               0  -A   B   B   B  -A   B|   36
                   0   B   B   B   B  -A|   36
                       0   B  -A  -A   B|   36
                           0   B  -A  -A|   36
                               0   B  -A|   36
                                   0   B|   36
                                       0|   36
```

Comments:

a) Pair sign ***+4.10.15*** mean, that graph consist of 10 vertices and 15 edges, that form *5-girths* – it is a ***complete invariant*** of Petersen's graph, by this sign cannot to construct an another graph. Pair sign *–2.3.2* fixate the paths between nonadjacent vertices of *5-girths.*

b) From **SM** can be read out, that in ***PET*** exist exactly twelve *5-girths*, in present case with vertices: 1-2-3-4-5-1 and 6-8-10-7-9-6 and 1-2-3-8-6-1 and 1-2-7-10-5-1 and 1-5-4-9-6-1 and 2-3-4-9-7-2 and 3-4-5-10-8-3 and 1-2-7-9-6-1 and 1-5-10-8-6-1 and 2-3-8-10-7-2 and 3-4-9-6-8-3 and 4-5-10-7-9-4. Each vertex belongs to six girths. Each edge belongs to four girths. It is ***5-girth-regular.***

c) In the complement ***PETC*** direct clique signs no exist, but pair graph of ***+2.5.8*** must contain a *4clique.*-klikki. In ***PETC*** are five *4-cliques*, in present case with vertices 1,3,9,10 and 1,4,7,8 and 2,4,6,10 and 2,5,8,9 and 3,5,6,7. Each vertex belongs to two cliques. Each edge belongs to one clique. It is ***4-clique-regularity.***

d) Semiotic model which correspond to pair graph with pair sign ***+2.5.8***, for example semiotic model $S_{1.3}$, that contain *4clique* sign ***+2.4.6***, and we see that it open the *4-clique* 1,3,9,10:

```
-A: -2.4.5;  B: +2.3.3;  C: +2.4.6;  D: +2.5.8.

   1   3|  9  10|  7|    i   ABCD    k    123
   0   D|  C   C|  B|    1   0121    1    121
       0|  C   C|  B|    3   0121    1    121
        |  0   C| -A|    9   1030    2    210
               0| -A|   10   1030    2    210
                   0|    7   2200    3    200
```

Proposition 2.4. The ***compliment*** of a symmetric (i.e. edge symmetric) *girth regular* graph is *clique regular*, and contrariwise.
Each vertex of a clique-(or girth)regular graph belong just to ***a*** cliques (or girths) and each edge just to ***b*** cliques (or girths).

NB! For characterizing a transitive graph is sufficient of ***its pair signs*** and ***frequency vectors***.

Example 2.2. First two rows of semiotic models of Heawood's graph ***HEA*** and its complement ***HEAC***:

A:-3.8.9; B:-2.3.2; ***C:+5.14.21.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14|	*i*	*AB*C	*k*	*deg*
0	**C**	*-B*	*-A*	*-B*	**C**	*-B*	*-A*	*-B*	*-A*	*-B*	*-A*	*-B*	**C**|	1	*46*3	*1*	3
	0	**C**	*-B*	*-A*	*-B*	*-A*	*-B*	*-A*	*-B*	**C**	*-B*	*-A*	*-B*|	2	*46*3	*1*	3

A:-2.10.36; ***B:+2.8.22; C:+2.9.30.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14|	*i*	*A*BC	*k*	*deg*
0	*-A*	**C**	**B**	**C**	*-A*	**C**	**B**	**C**	**B**	**C**	**B**	**C**	*-A*|	1	*3*46	*1*	10
	0	*-A*	**C**	**B**	**C**	**B**	**C**	**B**	**C**	*-A*	**C**	**B**	**C**|	2	*3*46	*1*	10

Comments:

a) Graph ***HEA*** is ***edge symmetric, 6-girth-regular*** and ***bipartite***, where its parts in present case divide to vertices with even numbers and vertices with odd numbers. It is also.

b) ***HEA*** is *structurally unique*, the pair sign ***+5.14.21*** signify that its 14 vertices form 21 adjacent pairs that belong to *6*-girths and form a ***complete invariant*** of this graph.

c) From bipartite ***HEA*** conclude that its complement ***HEAC*** consist of two ***mutually connected*** *7-**cliques***, it is ***7-clique regular***, where the cliques correspond to the parts of ***HEA. HEAC*** is also ***"non-edge" symmetric, 2-distance-*** and ***10-degree regular.***

Proposition 2.5. Complement of a ***m**-partite* graph in case of equal ***n*** parts is ***n-clique regular***, with the number ***m*** of non-intercrossed ***n-cliques.***

Example 2.3. First two rows of semiotic models of dodecahedron ***DOD*** and its complement ***DOCC***:

-A=-5.20.30; -B=-4.8.9; -C=-3.4.3; -D=-2.3.2; ***+E=+4.8.9.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20|	*i*	*ABCD*E	*k*
0	**E**	*-D*	*-C*	*-B*	*-C*	*-D*	**E**	*-D*	*-C*	*-B*	*-A*	*-B*	*-C*	*-D*	*-D*	*-C*	*-C*	*-D*	**E**|	1	*1366*3	*1*
	0	**E**	*-D*	*-C*	*-C*	*-D*	*-D*	*-C*	*-B*	*-A*	*-B*	*-C*	*-D*	**-E**	*-D*	*-C*	*-B*	*-C*	*-D*|	2	*1366*3	*1*

-A=-2.16.102; ***+B=+2.14.78; +C=+2.14.79; +D=+2.15.89.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20|	*i*	*A*BCCD	*k*
0	*-A*	**D**	**B**	**C1**	**B**	**D**	*-A*	**D**	**B**	**C1**	**C2**	**C1**	**B**	**D**	**D**	**B**	**B**	**D**	*-A*|	1	*3*6316	*1*
	0	*-A*	**D**	**B**	**B**	**D**	**D**	**B**	**C1**	**C2**	**C1**	**B**	**D**	*-A*	**D**	**B**	**C1**	**B**	**D**|	2	*3*6316	*1*

Comments:

a) If graph ***DOD*** is ***edge symmetric*** then its complement ***DODC*** is ***"non-edge" symmetric.***

b) ***DOD*** is ***5-girth-*** and ***3-degree regular*** and ***DODC*** is ***2-distance- and 16-degree regular.***

Is the complement of *5-girth-regular* dodecahedra ***DODC*** *clique-regular*?

In complement ***DODC*** the explicit clique signs no exist, but in the processing the pair graphs g_{ij}, for example with signs ***+B=+2.14.78*** obtained local semiotic models **SM**$_{1.4}$, **SM**$_{5.9}$, **SM**$_{3.16}$, **SM**$_{6.13}$ and **SM**$_{5.8}$, contain *8-clique signs* ***+2.8.28***. On the ground of such local semiotic models can be to recognize all the "hidden" ***partial 8-cliques*** of ***DODC***:

i=	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
I	●			●			●			●		●			●		●		●	
II		●			●		●		●		●			●			●			●
III	●		●			●			●			●		●		●		●		
IV		●		●		●		●			●		●			●			●	
V			●		●			●		●			●		●			●		●

Thus, the complement ***DODC*** is ***8-clique-regular***, where all five partial cliques are ***intercrossed***, and where all the 10 intercrossing edges belong to pair orbit ***C2***.

i-j=	1-12	2-11	3-18	4-19	5-20	6-16	7-17	8-13	9-14	10-15
Partial clique	**I**	**II**	**III**	**I**	**II**	**III**	**I**	**IV**	**II**	**I**
Partial clique	**III**	**IV**	**V**	**IV**	**V**	**IV**	**II**	**V**	**III**	**V**

Form known graphs are *clique regular* also complements of Coxeter's and Folkman's graphs. Their originals are bipartite and by all the nature laws represent the complements of such parts self-evidently cliques.

Proposition 2.6. Partial cliques of a clique regular graph can be ***disconnected partial, mutually connected*** or ***intercrossed.***

Intercrossing can be exist on the aspect of vertices and edges. For example: cliques of ***PETC*** intercrossed by vertices, of ***DODC*** by edges.

With this is the semiotic model as a text no exhausted (depleted).

2.2. Structural symmetry – positions

One of the key features of structure is ***symmetry.*** Symmetry is a structural characteristic that be expressed as a *recurrence* (in space or time) the similar parts (elements) of an object [Новая]. In this sense, represents symmetry an ***equivalence class***, which consists of "similar" elements, and they form a ***position*** in the structure. Position characterized by *isomorphism of its accompanying graphs*.

However, it is a widespread understanding of *symmetry* characteristic, where the parts (elements) take similar then, if these are located from a central point, or an axis on the same distance [Schmidt]. Such widespread is in mathematics defined as: a) the shape feature "transform to itself" (e.g. isometric); b) feature of binary relation ***xRy*** ↔ ***yRx***. Directed graphs called such a link (edge) as well as symmetrical.

Propositions 2.7. On the relationships between ***positions*** and ***adjacent graphs*** (D1.3). Let vertex ***i*** has adjacent vertices ***j*, j**, …*** and vertex ***j*** adjacent vertices ***i*, i**, … .***

P2.7.1. Vertices ***i, j, …*** have in graph ***G*** *the same* or *equal position* iff the *remain graphs are isomorphic* $(G_i=G\backslash v_i) \cong (G_j=G\backslash v_j) \cong ....$ and the *adjacent graphs* of their incident edges ***ij*, ji*, …*** are also isomorphic $(G^{adj}_{ij*}=G\backslash e_{ij*}) \cong (G^{adj}_{ji*}=G\backslash e_{ji*}) \cong ....$.

P2.7.2. Edges ***ij*, ji*, …*** have in graph ***G*** *the same* or *equal position* iff the *adjacent graphs are isomorphic* $(G^{adj}_{ij*}=G\backslash e_{ij*}) \cong (G^{adj}_{ji*}=G\backslash e_{ji*}) \cong \dots$ and the *remain graphs* of their incident vertices ***i, j, …*** are also isomorphic $(G_i=G\backslash v_i) \cong (G_j=G\backslash v_j) \cong \dots$.

These conditions show how the vertices and vertex pairs are mutually related.

General concept of symmetry is defined in mathematics by ***automorphism*** α, as a ***substitution which retains the structure***. It is treated also as an ***inner-*** or ***local isomorphism*** *(isomorphism with itself)*. Substitution has also been associated with renumbering of elements. In fact, none renumbering does not change the structure, it changes only the *removal, addition or relocation of a edge*. The automorphisms form the automorphism group of graph ***AutG*** where its *transitivity domains* to ***orbits*** Ω called. An orbit is practically the same *equivalence class* what coincide whit previous *position class.* In case of ***AutG*** be interested primarily on vertex orbits.

In the *frame of vertex positions* have also the *vertex pairs own positions* or *orbits*. Here is suitable issue from ***pair positions*** or ***-orbits.*** From now on we stay by term ***position (class)***.

Propositions 2.8. The relationships between ***automorphisms, local isomorphisms, transitivity domains, pair positions*** and ***pair signs***:

P2.8.1. As an *automorphism* α or permutation that retain the structure be expressed in the form of a *local isomorphism* $G^{adj}_{ij} \cong G^{adj}_{i*j*}$ then constitute *transitivity domain of automorphisms* or ***pair position(class)*** ΩR_n an *isomorphism class of adjacent graphs* $\{G^{adj}_{ij1} \cong G^{adj}_{ij2} \cong \dots \cong G^{adj}_{ijq}\}_n \subseteq \Gamma^{G}_{n}$.

A pair position ΩR_n, as *isomorphism class* Γ_n can be interpretable also as an “isomorphism clique”, where all the element pairs are mutually isomorphic.

P2.8.2. Isomorphism class of adjacent graphs Γ^{G}_{n} *is replaceable* with corresponding *isomorphism class of pair graphs* $\{g_{ij1} \cong g_{ij2} \cong \dots \cong g_{ijq}\}_n \subseteq \Gamma^{g}_{n}$, that as well characterize the vertex pairs. Identification the elements of position ΩR_n take place by *pair signs* $\pm d.n.q._{ij}$ (or $\pm d.n.q.x._{ij}$) as the identifiers of isomorphism class Γ_n.

P2.8.3. In the semiotic model ***SM*** is each vertex position ΩV_k related directly with pair positions ΩR_n of its incident edges.

The classes of vertices and vertex pairs in the semiotic model are in fact vertex- ΩV_k and pair positions ΩR_n correspondingly.

Conclusions 2.1. On the relations ***between modes of recognition the orbits (positions)*** by group theoretic and by semiotic modelling.

1) The vertex- and pair classes in semiotic models ***SM*** that showed on Examples 1.1. – 1.6., 2.1. – 2.3 and next correspond to ***vertex-*** ΩV_k and ***pair orbits (positions)*** ΩR_n.
2) The orbits, recognized by group theoretic orbits, and positions, recognized by semiotic modelling, ***coincide!***
3) Graphs with different structures can be have one and same group ***AutG***, but have different semiotic models ***SM.***
4) In case group theoretic treatment the number of permutations of completely symmetric graphs can be increase up to factorial. In case semiotic modelling of this does not happen.
5) In case group theoretic treatment the recognitions of vertex and edge orbits takes place separately and the “non-edge orbits” does not exist. In case semiotic modelling the recognitions of vertex-, pair(+)- and pair(-)orbits take place completely, where semiotic model ***SM*** express these in a complex.
6) Up to present considered, that orbit recognition belongs to periphery of graph theory. On the semiotic aspect it is a central problem.

Corollary 2.1. ***A position (equivalence class) and orbit is the same.***

Definitions 2.1. ***Symmetry signs*** of the structure*:*

D2.1.1 A vector with elements $|\Omega V|^m$, where $|\Omega V|$ is the power of a vertex position and m is the number of positions with such power, called ***sign of vertex symmetry SVV***.

D2.1.2. A vector with elements $|\Omega R|^m$, where $|\Omega R|$ is the power of a pair position and m is the number of positions with such power, called ***sign of pair symmetry SRV***.

Symmetry signs of the graphs on Examples 1.1 – 1.4 coincide, these are $SVV=1^1\,2^1\,3^1$, $SRV=1^1\,2^1\,3^2\,6^1$. The *edge symmetry* is here different, on Example 1.1 it is $SEV=1^1\,3^1\,6^1$.

Propositions 2.9. ***Measurement*** of symmetry. Symmetry signs give a good possibility to their *measuring*. To foundation of *symmetry size* is the classical Shannon's formula of ***information capacity***. Information capacity is practically a measure of *asymmetry or inner diversity*:

P2.9.1. ***Vertex information capacity HV*** depends from the number of vertices $|V|$ and the power of vertex positions $|\Omega V_k|$:

$$HV=-\sum_{k=1}^{K} PV_k \log PV_k,$$

where $0 \leq PV_k=|\Omega V_k|:|V| \leq 1$.

There $minHV=0 \leq HV \leq \log|V|=maxHV$, where, if $K=1$, then $HV=0$ and if $K=|V|$, then $HV=\log|V|$.

P2.9.2. ***Pair information capacity HR*** depends from the number of vertex pairs $|R|$ and the power of pair positions $|\Omega R_n|$:

$$HR=-\sum_{n=1}^{N} PF_n \log PF_n,$$

where $0 \leq PF_n=|\Omega R_n|:|R| \leq 1$ ja $|R|=[|V|(|V|-1)]:2$.

There $minHR=0 \leq HR \leq \log|R|=maxHR$, where, if $N=1$, then $HR=0$ and if $N=|R|$, then $HR=\log|R|$. Edge info capacity HR^+ calculates by the number of edges $|E|$ and the power of edge positions $|\Omega R_n^+|$. **c)** "Non-edge" info capacity HR^- calculate by the number of "non-edges" $|R^-|$ and the power of corresponding pair positions $|\Omega R_n^-|$.

Information comes into being on the ground of certain *diversity, i.e. distinguishing*. Information capacity depends from *quantity of variances*. There where variances no exist, arises a certain "domain of equability", what on the structural aspect a *symmetry class* or *orbit* mean. Then more exist "domains of equability" or orbits, then larger is information capacity ***HR*** and then smaller is the *symmetry size (value)*.

Proposition 2.10. On the ground of information capacities HV and HR can be recognize the ***symmetry values SV*** and ***SR*** correspondingly:

$$SR = 1 - (HR : \log|R|), \text{ where } 0 \leq SR \leq 1.$$

The symmetry ***value is 1***, if there exist *only one orbit*; the ***value is 0***, if the *number of orbits equal to the number of elements*. This give rise to *compare, order and grouping* the graphs with different size by symmetry values By analogy with the value of *pair symmetry* ***SR*** can be express the *pair(+)symmetry* (*"edge-symmetry"*) ***SE.*** Symmetry value *SR* is officially called as ***regularity.***

Symmetry-vectors and the symmetry-values of the graphs showed on Example 1.1.

Symmetry	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*	*SEV*	*SE*	*aut*	*3003PS*
Partial	3	5	$1^1 2^1 3^1$	*0.478*	$1^1 2^1 3^2 6^1$	*2.106*	*0.461*	$1^1 3^1 6^1$	*0.610*	12	*60*

Definitions 2.2. The ***symmetry kinds*** of graph structure:

D2.2.1. Graph with only *one* vertex position (orbit) ΩV_k we call ***vertex symmetric graph*** that also ***transitive*** called.

For transitive or vertex symmetric graphs:

D3.2.2. Transitive graph with only *one* pair position (orbit) ΩR^{+}_{n} is ***completely symmetric*** or ***complete graph.***

Empty graph with only one "non-edge" or pair position (orbit) is also *completely symmetric*.

D2.2.3. Transitive graph with only *one* edge position (i.e. pair(+)orbit) ΩR_{n}^{+} and only *one* "non-edge" position (i.e. pair(–)orbit) ΩR_{n}^{-} we call ***bisymmetric graph.***

For example, Petersen graph (Example 2.1) is bisymmetric.

D2.2.4. Transitive graph with *one* edge position (pair(+)orbit) ΩR_{n}^{+} and *any* "non-edge" positions (pair(–)orbits) ΩR_{n}^{-} we call ***edge symmetric*** or (+)***symmetric graph***.

Edge symmetric are here Heawood's (Example 2.2) and Dodecahedra (Example 2.3) and graphs, their number of different pair signs equal to the number of pair orbits. Complement of an *edge symmetric* graph is a ***"non-edge"-*** or (–)***symmetric graph***. Jointly we call these ***mono symmetric graphs.***

D2.5.5. Transitive graph with *any* edge positions (pair(+)orbits) ΩR_{n}^{+} and *any* "non-edge" positions (pair(–)orbits) ΩR_{n}^{-} we call ***poly-symmetric graph.***

Transitive graphs exist rarely. For example, among 156 of 6-elements structures exists such only 8. Also among the regular and strongly regular exist transitive rarely.

For non-transitive graphs:

D2.2.6. Graph with *more than one* vertex position ΩV_{k}, whereby at least to one ΩV_{k} belong at least two elements we call ***partially symmetric graph***.

Partially symmetric graph showed on Examples 1.1 – 1.6. Partial symmetry is a broad form of transition at symmetry to **a**symmetry. For example, among 156 of 6-elements structures are 140 partially symmetric.

D2.2.7. Graph where the number of vertices $|V|$ and vertex positions ΩV_{k} K is equal is a ***0-symmetric*** or *(completely)* ***asymmetric graph.***

A *0*-symmetric graph showed on Example 1.8. In case of little graphs is also an exceptional phenomenon. For example, among 156 of 6-elements structures are *0*-symmetric only 8.

2.3. Position structures

How can be represents a position? Very simply! A pair position ΩR_{n}, as a part of the formation of vertex pairs ***GS*** constitute also a graph. As a rule, the *position structures* open the "hidden sides" of graphs.

Definition 2.3. A *sign graph* G_{p}, whose edges e_{ij} correspond to elements of a certain pair position ΩR_{n} of ***G*** is a ***position graph*** G_{n} of ***G.***

Each position(class) ΩR_{n} can be represent as a graph G_{n}. Position graph is a self-evident attribute of a graph – it is a ***representation of an position(class) or orbit***. Each *position*(+)*graph* of ***G*** coincide with the corresponding *position*(–)*graph* of complement $\urcorner G$.

The main problem is there the *graph decomposition to its position structures* GS_{n}. Consider the decomposition a graph to their position structures.

Example 2.6. Semiotic model **SM** of Folkman's bipartite uncoordinated graph ***FOL*** and its position structures GS_{n}**:**

`A:-4.14.21; B:-3.8.10; C:-2.6.8; D:-2.4.4; E:-2.3.2;` **`F:+3.6.8.`**

1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2		u_i	k	s_i
11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	i	ABCDE**F**		12
0	-E	-E	-E	-E	-E	-E	-E	-E	-C	**F**	-B	-B	**F**	-B	**F**	-B	-B	-B	-F	**11**	06108**4**	1	04
	0	-E	-E	-E	-E	-E	-E	-C	-E	-B	**F**	-B	-B	**F**	-B	-B	-B	**F**	**F**	**12**	06108**4**	1	04
		0	-E	-E	-E	-E	-C	-E	-E	**F**	-B	**F**	-B	-B	-B	-B	**F**	**F**	**B**	**13**	06108**4**	1	04
			0	-E	-E	-C	-E	-E	-E	-B	**F**	-B	**F**	-B	-B	**F**	**F**	-B	-B	**14**	06108**4**	1	04
				0	-C	-E	-E	-E	-E	-B	-B	**F**	-B	**F**	**F**	**F**	-B	-B	-B	**15**	06108**4**	1	04
					0	-E	-E	-E	-E	-B	-B	**F**	-B	**F**	**F**	**F**	-B	-B	-B	**16**	06108**4**	1	04
						0	-E	-E	-E	-B	**F**	-B	**F**	-B	-B	**F**	**F**	-B	-B	**17**	06108**4**	1	04
							0	-E	-E	**F**	-B	**F**	-B	-B	-B	-B	**F**	**F**	-B	**18**	06108**4**	1	04
								0	-E	-B	**F**	-B	-B	**F**	-B	-B	-B	**F**	**F**	**19**	06108**4**	1	04
									0	**F**	-B	-B	**F**	-B	**F**	-B	-B	-B	**F**	**20**	06108**4**	1	04
										0	-A	-D	-D	-A	-D	-A	-D	-D	-D	**1**	36060**4**	2	40
											0	-A	-D	-D	-A	-D	-D	-D	-D	**2**	36060**4**	2	40
												0	-A	-D	-D	-D	-D	-D	-A	**3**	36060**4**	2	40
													0	-A	-D	-D	-D	-A	-D	**4**	36060**4**	2	40
														0	-D	-D	-A	-D	-D	**5**	36060**4**	2	40
															0	-D	-A	-A	-D	**6**	36060**4**	2	40
																0	-D	-A	-A	**7**	36060**4**	2	40
																	0	-D	-A	**8**	36060**4**	2	40
																		0	-D	**9**	36060**4**	2	40
																			0	**10**	36060**4**	2	40

Comments:

a) Graph ***FOL*** is ***partially symmetric*** (actually is it also *edge symmetric*) and it divide by pair orbits *–A, –B, –C, –D, –E and* ***F*** to six *position structures*:

b) To pair sign *–A* corresponds *position structure* **$FOL_{n:-A}$** is ***Petersen's graph*(!)**. This is presented also the in partial model **SM_{22}**, if there sign *–A* replace with Petersen sign ***+4.10.15*** and *–D* replace with sign *–2.3.2.*

c) To pair sign *–B* correspond *position structure* **$FOL_{n:-B}$** turns out to ***an other bipartite uncoordinated graph***, designed by A. Titov [Titov], which has likewise a position structure in *Petersen's form.*

d) To pair sign *–C* correspond *position structure* **$FOL_{n:-C}$** is a ***bisymmetric 10-componentic***, from *2-cliques* constituent graph. In present case has one vertex of *2*-clique even-number and other an odd number.

e) To pair sign *–D* correspond *position structure* **$FOL_{n:-D}$** is *complement of Petersen's graph.*

f) To pair sign *–E* correspond *position structure* **$FOL_{n:-E}$** is *complement of position structure* **$FOL_{n:-C}$**, i.e. ***2-quinta clique*** (see P2.15).

g) To pair sign **+*F*** correspond *position structure* **$FOL_{n:-F}$** is surely *Folkman's graph* self.

Apparently distributed the graphs to the "genetic groups", the study of which is a separate project. Current experience suggests that the 20-vertices edge symmetric bipartite graphs form such a group.

Propositions 2.11. Properties of the ***position structures*** **GS_n**:

P2.11.1. Position structures *open* its basic structure ***GS*** on the various aspects and *present its hidden properties*.

If the basic structure is partite or contains other components, as cliques, girths etc, then emerge the corresponding vertex complexes in the position structures in another form.

P2.11.2. Position structure **G_n** is *vertex symmetric,* i.e. "transitive".

P2.11.3. To each edge position (pair(+)orbit) **ΩR_n^+** corresponds an position structure **GS_n^+** and it constitute a partial graph of ***G.*** To each "non-edge" position (pair(–)orbit) **ΩR_n^-** corresponds a position structure **GS_n^-** and it constitute a partial graph of complement **⅂*G.***

P2.11.4. To each edge position **ΩR_n^+** of graph ***G*** correspond a "non-edge" position (pair(–)orbit) **ΩR_n^-** of complement ⅂***G***, such that their *position structures coincide*, **$GS_n \equiv$ ⅂GS_n.**

P2.11.5. Different position structures of a graph and/or position structures of different graphs can be *isomorphic* or *coincide*.

P2.11.6. A position structure ***GS_n*** of ***G*** can be turns out isomorphic with basic graph self, $GS_n \cong G$. For example, one position structure of cube is likewise a cube.

P2.11.7. If a pair sign includes all the vertices and edges of a graph, then this is the *complete invariant* of graph.

This is, of course, unmanageable, but unanswered hypothesis. Among well-known graphs have such pair signs, for example, Petersen's (Example 2.1) and Heawood's (Example 2.2), Coxeter's, cube-, octahedron- and other graphs. Such graphs are in principle designable by its pair signs.

We be interested also on position structures of position structures, i.e. on *high degree orbit graphs.*

Propositions 2.12. Properties of the ***high degree position structures***:

P2.12.1. A second or higher degree position structure can be isomorphic or coincide with its basic graph or with a low degree position structure.

P2.12.2. Higher degree position structures can be no open complementary "hidden sides", these begin recur. Depending on the complexity of the graph, it maybe occurs in the case of third or higher degree.

P2.12.3. Induction the high degree position structures is a *convergent process*, it close with a crop up a low degree or basic graph.

By help position structures can be find the same attributes of various structures. For example, Hypercube and Möbius-Kantor graph have some common position structures etc.

2.4. Bisymmetry, clique- and strong regularity

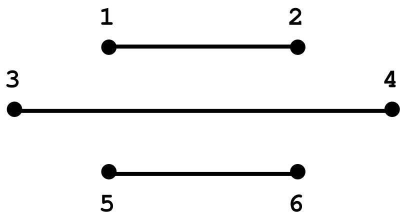
Interest for the relationship of regularity and symmetry is not seen, as in case of practical tasks these properties are usually not visible. However, it is with real legitimacy. An interesting relationship is between the bisymmetry and strong regularity, which seems to be "hidden" behind.

Proposition 2.13. The ***complement*** of a graph with ***m*** equal *disconnected partial cliques* is a ***bisymmetric m-partite complete graph***, i.e. it is a ***n-m-clique*** – and contrariwise.

Size	Graph	Its complement
m	Number of disconnected partial cliques	Number of parts
n	Power of disconnected partial cliques	Power of parts

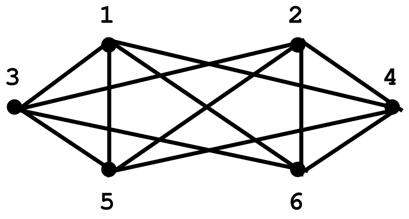
This mean that the complement of the structure with two disconnected partial cliques is a *bi-clique*, with three disconnected partial cliques is a *tri-clique,* etc. Not forget that the *complement* of bisymmetric graph is also bisymmetric.

Example 2.6. Bisymmetric **B6-3,** its complement **B6-12**, their semiotic models and common data:
:



```
              A:-0.2.0; B:+1.2.1.

| 1  2  3  4  5  6|  i   AB  deg
| 0  B -A -A -A -A|  1   41    1
     0  B -A -A -A|  2   41    1
        0  B -A -A|  3   41    1
           0  B -A|  4   41    1
              0  B|  5   41    1
                 0|  6   41    1
```



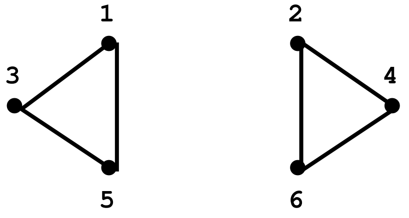
A:-2.6.12; **B:+2.4.5.**

1	2	3	4	5	6	*i*	*AB*	*deg*
0	-A	B	B	B	B	1	14	4
	0	-A	B	B	B	2	14	4
		0	-A	B	B	3	14	4
			0	-A	B	4	14	4
				0	-A	5	14	4
					0	6	14	4

SRV	*HR*	*SR*	*aut*
$3^1 12^1$	0.2173	0.8152	48

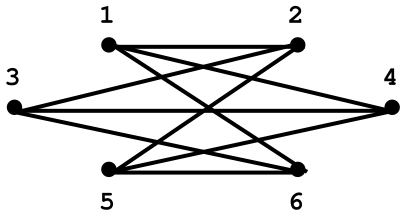
Comments **a)** Bisymmetric **B6-3** consist of ***three disconnected partial 2-cliques***, it is ***2-clique regular***. **b)** Bisymmetric complement **B6-12** is ***three partite***, where its parts correspond to 2-cliques of **B6-3**. It is a so called ***partite clique***, exactly with a ***2-tri-clique***, generally called ***n-m-clique***. It is simply sight that all the vertices belong to ***triangles***.

Example 2.7. Bisymmetric **B6-6,** its complement **B6-9**, their semiotic models and common data:



A:-0.2.0; **B:+2.3.3.**

1	2	3	4	5	6	*i*	*ABC*	*deg*
0	-A	B	-A	B	-A	1	32	2
	0	-A	B	-A	B	2	32	2
		0	-A	B	-A	3	32	2
			0	-A	B	4	32	2
				0	-A	5	32	2
					0	6	32	2



A: -2.5.6; **B:+3.6.9.**

1	2	3	4	5	6	*i*	*AB*	*deg*
0	B	-A	B	-A	B	1	23	3
	0	B	-A	B	-A	2	23	3
		0	B	-A	B	3	23	3
			0	B	-A	4	23	3
				0	B	5	23	3
					0	6	23	3

SRV	*HR*	*SR*	*aut*
$6^1 9^1$	0.2923	0.7515	72

Comments **a)** Bisymmetric **B6-6** consist of ***two disconnected partial 3-cliques***, it is ***3-clique regular***. **b)** Complement **B6-9** is ***bipartite***, where its parts correspond to 3-cliques of **B6-6. d) B6-9** is a ***3-bi-clique*** that is ***4-girth regular.*** **e)** Pair sign ***+3.6.9*** cover all the n=6 vertices and all the q=9 edges, it is the ***complete invariant*** of **B6-9**.

Propositions 2.14. On ***n-m-cliques*** and ***bisymmetry.***

P2.14.1. All the ***n-m-cliques*** with equal power ***n*** and their complements are ***bisymmetric.*** Bisymmetric can be connected or disconnected.
P2.14.2. A ***n-m-clique*** contain an (usual)clique with the power ***m***, it is ***m-clique regular.*** For example, bi-clique is *2-clique regular,* tri-clique is *3-clique regular,* etc.
P2.14.3. A bisymmetric ***n-m-clique*** contain $s=n^m$ *usual cliques* with power ***m.***

P2.14.4. The number of edges |***E***| of bisymmetric ***n-m****-clique* equal to the product of quadrate of power n^2 of parts and the number ***m*** of edges in the usual clique:

n-m-clique			
Symmetry	Power of cliques	Number of cliques	Number of edges
Bisymmetry	m	n^m	$E=n^2m(m-1):2$

Proposition 2.15. Correspondingly to the number of parts we call a ***n-m-clique*** to ***bi-, tri-, quadro-, quinta-, sexta-, septa-, octa-, nona-, deca-,*** or ***undeca-*** etc ***-clique.***

Among the graphs with 1 to 20 vertices there exist exactly following ***n-m-cliques***:

a) One *n-m-clique* with 4-vertces – ***2-biclique*** as a complement of disconnected partial 2-cliques;
b) Two *n-m-cliques* with 6 vertices – ***2-tri-clique*** and ***3-bi-clique*** as complements of disconnected partial 2- and 3-cliques correspondingly;
c) Two *n-m-cliques* with 8 vertices – ***2-quadro-clique*** and ***4-bi-clique*** as complements of disconnected partial 2- and 4-cliques correspondingly;
d) One *n-m-clique* with 9 vertices – ***3-tri-clique*** as a complement of disconnected partial 3-cliques correspondingly;
e) Two *n-m-cliques* with 10 vertices – ***2-quinta-*** and ***5-bi-clique*** as complements of disconnected partial 2- and 5-cliques correspondingly;
f) Four *n-m-cliques* with 12 vertices – ***2-sexta-, 3-quadro-, 4-tri-*** and ***6-bi-clique*** as complements of disconnected partial 2-, 3-, 4- and 6-cliques correspondingly;
h) Two *n-m-cliques* with 14 vertices – ***2-septa-*** and ***7-bi-clique*** as complements of disconnected partial 2- and 7-cliques correspondingly;
i) Two *n-m-cliques* with 15 vertices – ***3-quinta-*** and ***5-tri-clique*** as complements of disconnected partial 3- and 5-cliques correspondingly;
j) Three *n-m-cliques* with 16 vertices – ***2-octa-, 4-quadro-*** and ***8-bi-clique*** as complements of disconnected partial 2-, 4- and 8-cliques correspondingly;
k) Four *n-m-cliques* with 18 vertices – ***2-nona-, 3-sexta-, 6-tri-*** and ***9-bi-clique*** as complements of disconnected partial 2-, 3-, 6- and 9-cliques correspondingly;
l) Four *n-m-cliques* with 20 vertices – ***2-deca-, 4-quinta-, 5-quadro-*** and ***10-bi-clique*** as complements of disconnected partial 2-, 4-, 5- and 10-cliques correspondingly;

In all **27 *n-m-cliques***.

A graph said ***strongly regular*** with parameters (*k,**a,b***) if it is a *k*-regular incomplete and connected graph such that any two adjacent vertices have exactly $a\geq0$ common neighbors and any two non-adjacent vertices have $b\geq1$ common neighbors.

Proposition 2.16. All the *connected bisymmetric* structures are ***strongly regular*** as well ***girth-*** or ***clique regular***.

Existence in connected bisymmetric structure exactly two different pair signs, $-d.n_1.q$ and $+d.n_2.q$, mean that by $\pm d=2$ *has each nonadjacent vertex pair exactly* n_1***-2*** *common neighbors* and *each adjacent vertex pair* n_2***-2*** *common neighbors*. In case $+d>2$ no exist common neighbors. The numbers ***n-2*** of common neighbors can be stay constant also by existence more that two pair signs. Consequently, strongly regular graphs can be also *mono-, poly- and partial symmetric.*

Conclusion 2.2. All the ***n-m-cliques*** are *strongly regular*, but no on the contrary.

Proposition 2.17. The ***connected complement*** of a strongly regular structure is also *strongly regular*.

In addition of simply constructable ***n-m-cliques*** are recognized following bisymmetric-strongly regular structures: **1)** self-complemented 5-girth; **2)** self-complemented B9-18; **3)** Petersen graph ***PET*** (B10-15); **4)** and its complement ***PETC*** (B10-30); **5)** self-complemented B13-39; **6)** Weisfeiler's B15-45; **7)** and its complement B15-60; **8)** Greenwood's (Clebish's) B16-40; **9)** and its complement B16-80; **10)** Shrikhande B16-48; **11)** and its complement B16-72; **12)** self-complemented B17-68.

Conclusion 2.3. Among the graphs with 20 vertices exist 27+12= ***39 bisymmetric, strongly regular, clique- or girth regular*** structures [Tevet 38] .

Example 2,8. List of all the 39 *bisymmetric* + *strongly regular* + *clique-* or *girth regular* graphs with 1 to 20 vertices:

Nr	Notation	*deg*	*SRV*	*SR*	Cmp/prt		Regu-larity	Number *s*	Commentary	Pair signs	
					m	n				Pair(-)sign	Pair(+)sign
1	**B4-4**	2	$2^1 4^1$	***0.6448***	**2p**	2	*4-girth*	-	***2-bi-clique***	*–2.4.4*	***+3.4.4***
2	**B5-5**	2	5^2	***0.6990***	**1c**	5	*5-girth*	-	***Selfcomplem.***	*–2.3.2*	***+4.5.5***
3	**B6-12**	4	$3^1 12^1$	***0.8152***	**3p**	2	*3-clique*	8	***2-tri-clique***	*–2.6.12*	***+2.4.5***
4	**B6-9**	3	$6^1 9^1$	***0.7515***	**2p**	3	*4-girth*	-	***3-bi-clique***	*–2.5.6*	***+3.6.9***
5	**B8-24**	6	$4^1 24^1$	***0.8769***	**4p**	2	*4-clique*	16	***2-quadro-clique***	*–2.8.24*	***+2.6.13***
6	**B8-16**	4	$12^1 16^1$	***0.7906***	**2p**	4	*4-girth*	-	***4-bi-clique***	*–2.6.8*	***+3.8.16***
7	**B9-27**	6	$9^1 27^1$	***0.8431***	**3p**	3	*3-clique*	27	***3-tri-clique***	*–2.8.21*	***+2.5.7***
8	**B9-18**	4	18^2	***0.8066***	**3p**	3	*3-girth*	6	***Selfcomplem.***	*–2.4.4*	***+2.3.3***
9	**B10-40**	8	$5^1 40^1$	***0.9084***	**5p**	2	*5-clique*	32	***2-quinta-clique***	*–2.10.40*	***+2.8.25***
10	**B10-15**	3	$15^1 30^1$	***0.8328***	**1c**	10	*5-girth*	12	***Petersen gr.***	*–2.3.2*	***+4.10.15***
11	**B10-30**	6			**1c**	10	*4-clique*	5	***Petersen comp.***	*–2.6.12*	***+2.5.8***
12	**B10-25**	5	$20^1 25^1$	***0.8196***	**2p**	5	*4-girth*	-	***5-bi-clique***	*–2.7.10*	***+3.10.25***
13	**B12-60**	10	$6^1 60^1$	***0.9273***	**6p**	2	*6-clique*	64	***2-sexta-clique***	*–2.12.60*	***+2.10.41***
14	**B12-54**	9	$12^1 54^1$	***0.8868***	**4p**	3	*4-clique*	81	***3-quadro-clique***	*–2.11.45*	***+2.8.22***
15	**B12-48**	8	$18^1 48^1$	***0.8601***	**3p**	4	*3-clique*	64	***4-tri-clique***	*–2.10.32*	***+2.6.9***
16	**B12-36**	6	$30^1 36^1$	***0.8355***	**2p**	6	*4-girth*	-	***6-bi-clique***	*–2.8.12*	***+3.12.36***
17	**B13-39**	6	39^2	***0.8409***	**1c**	1	*3-clique*	22	***Selfcomplem.***	*–2.5.7*	***+2.4.5***
18	**B14-84**	12	$7^1 84^1$	***0.9399***	**7p**	2	*7-clique*	128	***2-septa-clique***	*–2.14.84*	***+2.12.61***
19	**B14-49**	7	$42^1 49^1$	***0.8470***	**2p**	7	*4-girth*	-	***7-bi-clique***	*–2.9.14*	***+3.14.49***
20	**B15-90**	12	$15^1 90^1$	***0.9119***	**5p**	3	*5-clique*	243	***3-quinta-clique***	*–2.14.78*	***+2.11.46***
21	**B15-75**	10	$30^1 75^1$	***0.8711***	**3p**	5	*3-clique*	125	***5-tri-clique***	*–2.12.45*	***+2.7.11***
22	**B15-45**	6	$45^1 60^1$	***0.8533***	**1c**	15	*3-clique*	-	***Weisfeiler***	*–2.5.6*	***+2.3.3***
23	**B15-60**	8			**1c**	15	*5-clique*	-	***Weisfeil. comp.***	*–2.6.12*	***+2.6.12***
24	**B16-112**	14	$8^1 112^1$	***0.9488***	**8p**	2	*8-clique*	256	***2-octa-clique***	*–2.16.112*	***+2.14.85***
25	**B16-96**	12	$24^1 96^1$	***0.8955***	**4p**	4	*4-clique*	256	***4-quadro-clique***	*–2.14.72*	***+2.10.33***
26	**B16-40**	5	$40^1 80^1$	***0.8670***	**4p**	4	*4-girth*	-	***Greenwood***	*–2.4.4*	***+3.10.13***
27	**B16-80**	10			**1c**	16	*5-clique*	16	***Greenw. comp.***	*–2.8.24*	***+2.8.22***
28	**B16-48**	6	$48^1 72^1$	***0.8594***	**1c**	16	*4-clique*	-	***Shrikhande***	*–2.4.4*	***+2.4.6***
29	**B16-72**	9			**1c**	16	*4-clique*	-	***Shrikhan comp.***	*–2.8.18*	***+2.6.11***
30	**B16-64**	8	$56^1 64^1$	***0.8557***	**2p**	8	*4-girth*	-	***8-bi-clique***	*–2.10.10*	***+3.16.64***
31	**B17-68**	8	68^2	***0.8589***	**1c**	17	*3-clique*	-	***Selfcomplem.***	*–2.6.11*	***+2.5.7***
32	**B18-144**	16	$9^1 144^1$	***0.9555***	**9p**	2	*9-clique*	512	***2-nona-clique***	*–2.18.144*	***+2.16.113***
33	**B18-135**	15	$18^1 135^1$	***0.9280***	**6p**	3	*6-clique*	729	***3-sexta-clique***	*–2.17.120*	***+2.14.79***
34	**B18-108**	12	$45^1 108^1$	***0.8796***	**3p**	6	*3-clique*	216	***6-tri-clique***	*–2.14.60*	***+2.8.13***
35	**B18-81**	9	$72^1 81^1$	***0.8626***	**2p**	9	*4-girth*	-	***9-bi-clique***	*–2.11.18*	***+3.18.81***
36	**B20-180**	18	$10^1 180^1$	***0.9607***	**10p**	2	*10-clique*	1036	***2-deca-clique***	*–2.20.180*	***+2.18.45***
37	**B20-160**	16	$30^1 160^1$	***0.9169***	**5p**	4	*5-clique*	1924	***4-quinta-clique***	*–2.18.128*	***+2.14.73***
38	**B20-150**	15	$40^1 150^1$	***0.9019***	**4p**		*4-clique*	625	***5-quadro-clique***	*–2.17.105*	***+2.12.46***
39	**B20-100**	10	$90^1 100^1$	***0.8682***	**2p**	10	*4-girth*	-	***10-bi-clique***	*–2.12.20*	***+3.20.100***

<u>Comments:</u> **a)** The marking of structure show the numbers of vertices and edges. **b)** ***deg*** – degree. **c)** ***SRV*** – symmetry vector (Def. 2.1.2); **d)** ***SR*** – symmetry value (Prop. 2.4). **e)** ***c*** **–** number of components. **f)** ***m*** – number of parts. **g)** ***n*** – power of parts. **h)** ***s*** – number of cliques.

So it is recognized 39 bisymmetric-strongly regular structures with 4 to 20 vertices, mainly on the ground of disconnected partial cliques induced. The results of J. Petersen (B10-15)**,** A. Titov (B13-39), B. Weisfeiler (B15-45), Greenwood-Gleason-Clebish (B16-40) in the realm of bisymmetry are random coincides, because the first be interested on valence-regularity, other on self-complementary, third on strong regularity, fourth on color-conjecture, others on isomorphism testing etc.

The lists of strongly regular graphs are incomplete. For example, in a special list [Strongly I] be lacking 31 strongly regular graphs with to 20 vertices. A "most complete" list [Strongly II] where be given 33 structures, among these also ***n-m-cliques*** fail unfortunately **25** (B16-96), **29** (B16-72), **33** (B18-135), **34** (B18-108), **37** (B20-160) and **38** (B20-150).

In the "most complete" list of strongly regular graphs are showed all the to 20 vertices ***bi-cliques***, as *complete bipartite graphs*, whereby bi-clique with 4 vertices called *square* and with 6 vertices called *unity.* There are also showed all the ***2-m-cliques***, that have title ***r-****cocktail party graphs*, whereby with 6 vertices called *octahedral graph* and with 8 vertices *16-cell graph.* Other ***n-m-cliques*** called mostly *circular graphs*. There lack five ***n-m-cliques*** and the complement of a known strongly regular graph.

There exist also many large graphs. For example, On list [Strongly I] to find a graph with 999 vertices:

16	(999, 448, 172, 224)	-	-

<u>Example 2.9.</u> It can in a simple way to induce some bisymmetric, clique- and strongly regular graphs with 999 vertices. In the lists of strongly regular graphs cannot these to find:

Nr	**Notation**	***deg***	***|E|***	***SR***	**Regularity**	**Commentary**	**(+)signs**
1	**B999-2**	2	999		*3-clique*	333 disconnected partial 3-cliques	***+2.3.3***
2	**B999-996**	996	497502	***0.9989***	*333-clique*	333 3-elementic parts ***3-tricent-triginta-tri-clique***	**?**
3	**B999-8**	8	3996		*9-clique*	111 disconnected partial 9-cliques	***+2.9.36***
4	**B999-990**	990	494505	***0.9979***	*111-clique*	111 9-elementic parts ***9-cent-undeca-clique***	**?**
5	**B999-110**	110	54945		*111-clique*	9 disconnected partial 111-cliques	***+2.111.6105***
6	**B999-888**	888	443556	***0.9736***	*9-clique*	9 111-elementic parts ***111-nona-clique***	**?**
7	**B999-332**	332	165832		*333-clique*	3 disconnected partial 333-cliques	***+2.333.55278***
8	**B999-666**	666	332667	***0.9515***	*3-clique*	3 333-elementic parts ***333-tri-clique***	**?**

<u>Comments:</u> **a)** Strongly regular are there only ***n-m-***cliques. **b)** The names of ***n-m-***cliques can be for any no please, but others I cannot find.

With very important symmetry properties is graph **B16-40** was constructed by Greenwood-Gleason as in any 3-colouring of the edges of the **K_{16}** without monochromatic triangles, the set of edges of each colour from this graph. It called also Clebish graph.

<u>Example 2.10.</u> Semiotic models and comments of bisymmetric strongly regular **B16-40** and its complement **B16-80**:

A:-2.4.4; **B:+3.10.13.**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	*i*	*A***B**	*deg*
0	**B**	-A	-A	**B**	-A	-A	**B**	-A	-A	-A	**B**	-A	-A	**B**	-A	**1**	105	5
	0	**B**	-A	-A	-A	**B**	-A	-A	**B**	-A	-A	-A	**B**	-A	-A	**2**	105	5
		0	**B**	-A	**B**	-A	-A	**B**	-A	-A	**B**	-A	-A	-A	-A	**3**	105	5
			0	**B**	-A	-A	**B**	-A	-A	**B**	-A	-A	**B**	-A	-A	**4**	105	5
				0	**B**	-A	-A	-A	**B**	-A	-A	**B**	-A	-A	-A	**5**	105	5
					0	**B**	-A	-A	-A	-A	-A	-A	-A	**B**	**B**	**6**	105	5
						0	**B**	-A	-A	**B**	-A	**B**	-A	-A	-A	**7**	105	5
							0	**B**	-A	-A	-A	-A	-A	-A	**B**	**8**	105	5
								0	**B**	-A	-A	**B**	-A	**B**	-A	**9**	105	5
									0	**B**	-A	-A	-A	-A	**B**	**10**	105	5
										0	**B**	-A	-A	**B**	-A	**11**	105	5
											0	**B**	-A	-A	**B**	**12**	105	5
												0	**B**	-A	-A	**13**	105	5
													0	**B**	**B**	**14**	105	5
														0	-A	**15**	105	5
															0	**16**	105	5

A:-2.8.24; **B:+2.8.22.**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	*i*	*A* **B**	*deg*
0	-A	**B**	**B**	-A	**B**	**B**	-A	**B**	**B**	**B**	-A	**B**	**B**	-A	**B**	**1**	510	10
	0	-A	**B**	**B**	**B**	-A	**B**	**B**	-A	**B**	**B**	**B**	-A	**B**	**B**	**2**	510	10
		0	-A	**B**	-A	**B**	**B**	-A	**B**	**B**	-A	**B**	**B**	**B**	**B**	**3**	510	10
			0	-A	**B**	**B**	-A	**B**	**B**	-A	**B**	**B**	-A	**B**	**B**	**4**	510	10
				0	-A	**B**	**B**	**B**	-A	**B**	**B**	-A	**B**	**B**	**B**	**5**	510	10
					0	-A	**B**	**B**	**B**	**B**	**B**	**B**	**B**	-A	-A	**6**	510	10
						0	-A	**B**	**B**	-A	**B**	-A	**B**	**B**	**B**	**7**	510	10
							0	-A	**B**	**B**	**B**	**B**	**B**	**B**	-A	**8**	510	10
								0	-A	**B**	**B**	-A	**B**	-A	**B**	**9**	510	10
									0	-A	**B**	**B**	**B**	**B**	-A	**10**	510	10
										0	-A	**B**	**B**	-A	**B**	**11**	510	10
											0	-A	**B**	**B**	-A	**12**	510	10
												0	-A	**B**	**B**	**13**	510	10
													0	-A	-A	**14**	510	10
														0	**B**	**15**	510	10
															0	**16**	510	10

Common invariants and measures of graph and its complement:

Symmetry	*|V|*	*|R|*	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*
Bisymmetry	**16**	**120**	**1**	**2**	16^1	1.000	$40^1 80^1$	0.2762	**0.8670**

Distinguishing invariants and measures:

G	*|E|*	*k*	N^+	N^-	*P*	*CL*	*MC*	*DM*	SEV^+	SE^+	*TRA*	*BRA*
B16-40	40	1	1	1	2	2	4	2	40^1	1.000	0	0
B16-80	80	1	1	1	2	5	3	2	80^1	1.000	1.000	0

Comments: **a)** The ***bisymmetric*** and ***strongly regular*** structure **B16-40** is correspondingly to pair(+)sign ***+3.10.13 4-girth regular***, that mean *partition.* This appear also to ***4-partite*** with incompletely connected parts on *4-elementical bases*. **b)** It no quadroclique. **c)** The parts are *variety*, where, for example one variant is **A=5,8,12,15; B=3,7,10,14**; **C=1,4,9,16**; and **D=2,6,11,13**:

	A	**B**	**C**	**D**
A	0	4	6	10
B		0	10	6
C			0	4
D				0

d) From 4-elemintic parts of **B16-40** conclude the ***4-clique regularity*** of variety cliques of complement **B16-80**. **e)** On the other hand, in case of each vertex of **B16-40** its 5 adjacent vertices no have between themselves adjacencies (edges), from which conclude also a ***5-clique-regularity*** of complement **B16-80**. We can in **B16-80** to fix 16 different 5-cliques, such as (beginning at the adjacent vertices of first vertex of **B16-40**) **2,5,8,12,15**; **1,3,7,10,14**; … to ending with **6,8,10,12,14**.

Next chapter: 3. Semiotic model and isomorphism problem