SEMIOTIC MODELLING OF THE GRAPHS
(A review with some examples)

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Semiotic Modelling of the Graphs (called also Structure Semiotics) is a domain of research on the frontiers of ***graph theory*** and ***semiotics***, which investigates the abstract concept of ***structure***.

Under the *structure* be understand the general as well as the cognitive (epistemological) meaning of the structure as a relationship or organizational form of its elements [Schmidt] [Новая]. Structure is presentable in the form of a ***graph*** and is intimately related with *invariance* and *isomorphism.* Semiotics of the structure is one of the many kinds of object-oriented semiotics.

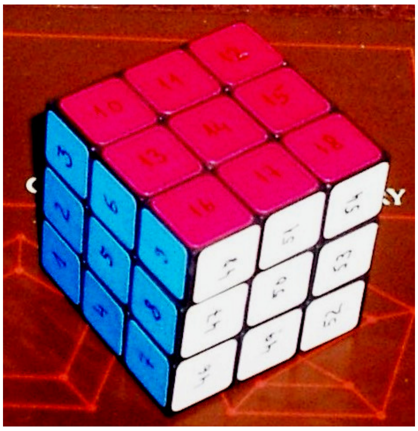
Objectives

- Exploring the general meaning of structure and compilation its ***formalized interpretation***.
- Studying the ***structural signs***, elaboration their system and corresponding ***processing algorithms*.**
- Representation the structure and its properties in the ***canonical form***.
- Exploring the ***structural properties***.

Principles

In each *system* have an important role their *empirical properties* of elements and relationships. Each system has a *function* and *structure*. Structure constitutes an ***abstraction of the system***, its "skeleton", where its elements and relationships are loose empirical meanings and their diversity is expressed in the form of different ***positions*** in the structure.

Example 1. Concepts of ***system, structure, position and graph*** are easily and pictorially explainable on the *Rubik's Cube.*



Comments: **a)** In Rubik's cube has in each facet one element in the *middle*, four elements in the *edges* and four elements in the *angles*. Thus, the 6 elements of the cube represent a ***"middle position"***, 24 elements an ***"edge position"*** and 24 elements an ***"angle position"***. **b)** With turning the layers of the cube, although be ***changed the system,*** because the relationships between its empirical properties of the elements (i.e. colors) changes. However, the ***structure does not change***, because the ***positions are remain.*** **c)** The structure of Rubic's cube is presentable in the form of the graph with ***three vertex positions***.

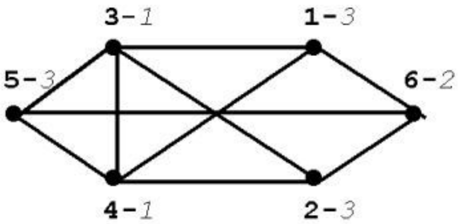
The system theoretical concept of position coincides with the concept of ***orbit*** in graph theory. Isomorphic graphs have one and the same structure. Ascertainment of *isomorphism* does not mean the recognition of structure, it means only a determination of the *equivalence* of structures. ***Recognition*** of the structure involves its ***description*** with structural signs.

Realization

Structural signs are the specific invariants of vertex pairs, known as ***pair signs*** in the form of a quadruplet ±***d.n.q.***$_{ij}$, where ***+d*** show collateral- and ***–d*** ordinary distance between vertices $\boldsymbol{v_i}$ and $\boldsymbol{v_j}$, ***n*** – number of vertices and ***q*** – number of edges, in this *pair graph* $\boldsymbol{g_{ij}}$ (see Chapter **1.1**).

Based on pair signs be constructed the ***semiotic model*** **SM** in the form of *canonical sign matrix S* what *describe* the structure. Study of the structure is equivalent to a study of its semiotic model (text of structure). The number of different structures is equal to the number of non-isomorphic graphs. Ascertainment of the identity of structures constitutes a simple method of ascertaining the equivalence of their respective sign matrices.

Example 2. Two graphs and their semiotic models:



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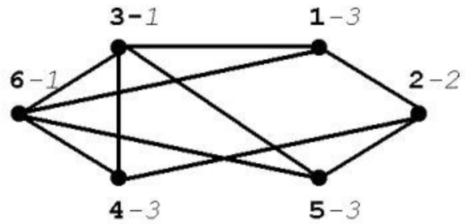


A:-2.5.7; B:-2.5.6;
C:+2.3.3; D:+2.5.7; E:+3.6.10.

1	1	2	3	3	3		u_i	k	s_i
3	4	6	1	2	5	i	ABCDE		123
0	D	-B	C	C	C	3	01310	1	103
	0	-B	C	C	C	4	01310	1	103
		0	E	E	E	6	02003	2	003
			0	-A	-A	1	20201	3	210
				0	-A	2	20201	3	210
					0	5	20201	3	210

A:-2.5.7; B:-2.5.6;
C:+2.3.3; D:+2.5.7; E:+3.6.10.

1	1	2	3	3	3		u_i	k	s_i
3	6	2	1	4	5	i	ABCDE		123
0	D	-B	C	C	C	3	01310	1	103
	0	-B	C	C	C	6	01310	1	103
		0	E	E	E	2	02003	2	003
			0	-A	-A	1	20201	3	210
				0	-A	4	20201	3	210
					0	5	20201	3	210

Comments: **a)** Different graphs have here *equivalent sign matrices*, i. e. the ***structures are equivalent*** and corresponding ***graphs are isomorphic***. **b)** Sign matrix be recognize ***three vertex positions (-orbits)*** and ***five orbits of vertex pairs***, including two "non-edge" orbits. **c)** The correspondence between structures be expressed on the level of orbits of vertex pairs. **d)** The pair signs ascertain for each vertex pair its connectivity mode, its belonging to a girth or clique with fixed size and so on, for example, ***E: +3.6.10*** means: "the vertex pair belongs to more than one girth with length *d= 4*"(see Chapter **2.1**). **e)** In case of symmetric structures used complementary *adjusted pair signs* (Prop. **1.1**).

The semiotic approach opens up the hitherto little known or unnoticed structural qualities. For this aim used the ***accompanying graphs*** (Chapter **1.1**), such as:

- **a)** *Complement* ⅂***G***.
- **b)** *Pair graph* $\boldsymbol{g_{ij}}$, as intersection $\boldsymbol{N_i \cap N_j}$ of neighborhoods of the vertex pair ***ij.***
- **c)** *Sign graph* $\boldsymbol{G_p}$, as a graph, where its edges correspond to a sign class ***p*** of semiotic model.

d) *Adjacent graphs* $\boldsymbol{G^{adj}}$, The *greatest sub-graph* $\boldsymbol{G^{sub}} = \boldsymbol{G \backslash e_{ij}}$ and *smallest super-graph* $\boldsymbol{G^{super}} = \boldsymbol{G \cup e_{ij}}$ of ***G***.

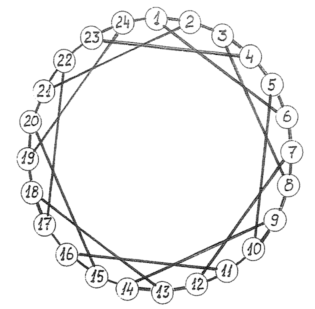
The structure be studied (investigates) *in an integrated way*, in conjunction with its *complement*. *Pair graph* be characterize the condition of a vertex pair, *pair sign* is only its invariant. In some cases, it is necessary to open the *semiotic model of pair graph* for adjustment of corresponding pair sign (Prop. **1.1.2**). *Sign graph* is one of the key attributes of structure, by it can *adjust the pair signs* (Prop. **1.1.3**) and in certain cases investigate the *structural properties* (Chapter **2.3**).

The semiotic model constitutes a ***text of structure*** (Chapter **2**), what make possible to explore the ***regularities***, such as *distance-, girth-, clique- and strong regularity* (Chapter **2.1**). The most important characteristic of structure is its ***symmetry*** (Chapter **2.2**). Symmetry properties, i. e. the *orbits (positions)*, are recognizable in sign matrix as the equivalence classes of pair signs. Thus, a simple method replaces the conventional method of studying the full ***automorphism group AutG***. The *vertex orbits* as well as the *vertex-pair orbits* are easily recognizable, including the last of the *edge-* and *"non-edge"* orbits. The *classification of symmetry kinds (properties)* has been developed on the grounds of *symmetry characteristics* (the number of orbits and their power) the. This provides a way to *measure* the symmetry and also the asymmetry of the structure.

To each orbit of a vertex pair corresponds a ***position structure*** (Chapter **2.3**). The position structure is a *sign graph,* be formed on the basis of the vertex pairs of its orbit. It is a tool for study the "hidden" structural properties. For example, one of position structures of Folkman graph is Petersen graph, etc. Be explored also the relationships between *bisymmetry, clique-* and *strong regularities* (Chapter **2.4).** This approach is successful in studying the problems of *bi-, tri- etc. partite structures*.

Semiotic model is also the ***canonical submission*** of a graph (Chapter **3.1**). In common cases are the structures recognizable on the level of initial pair signs, but in case of some symmetric graphs are necessary to use the ***adjusted pair signs***.

<u>Example 3.</u> Transitive graph ***G***, its initial pair signs, adjustable pair signs and the complete sign matrix:



A:-5.18.23; B:-4.9.10; C:-4.8.8; D:-4.7.7; E:-3.8.9; F: -3-3-6; G:-3.4.3; H:-2.3.2;
I:+5.10.12; J:+5.12.15; K:+5.14.18.

Initial *pair signs* $\boldsymbol{dnq_{ij}}$ of ***G***, their markings ***p*** and ordering numbers ***n***, ***adjustable*** *pair signs* $\boldsymbol{dnq_{ij}^{p=F}}$ by sign graph $\boldsymbol{G_{p=-F}}$, markings of ***adjusted*** *pair signs* ***p**** and ordering numbers ***n*** of pair positions:

dnq_{ij}	p	n	$dnq_{ij}^{p=-F}$	p*	n*
-5.18.23	*-A*	**1**	*-5.10.12*	*A1*	**1**
			-5.8.8	*A2*	**2**
-4.9.10	*-B*	**2**	*-4.7.7*	*B*	**3**

−4.8.8	−C	**3**	−2.4.4	C	**4**
−4.7.7	−D	**4**	−2.3.2	D	**5**
−3.8.9	−E	**5**	−3.10.12	E	**6**
−3.6.6	−F	**6**	+3.4.4	F1	**7**
			+5.8.10	F2	**8**
			+3.4.4	F3	**9**
−3.4.3	−G	**7**	−3.8.10	G1	**10**
			−3.6.6	G2	**11**
			−3.4.3	G3	**12**
−2.3.2	−H	**8**	−6.20.26	H1	**13**
			−4.7.7	H2	**14**
			−2.3.2	H3	**15**
+5.10.12	***I***	***9***	−3.6.6	***I***	***16***
+5.12.15	***J***	***10***	−3.4.3	***J***	***17***
+5.14.18	***K***	***11***	−5.8.8	***K***	***18***

The complete semiotic model **SM** of ***G***:

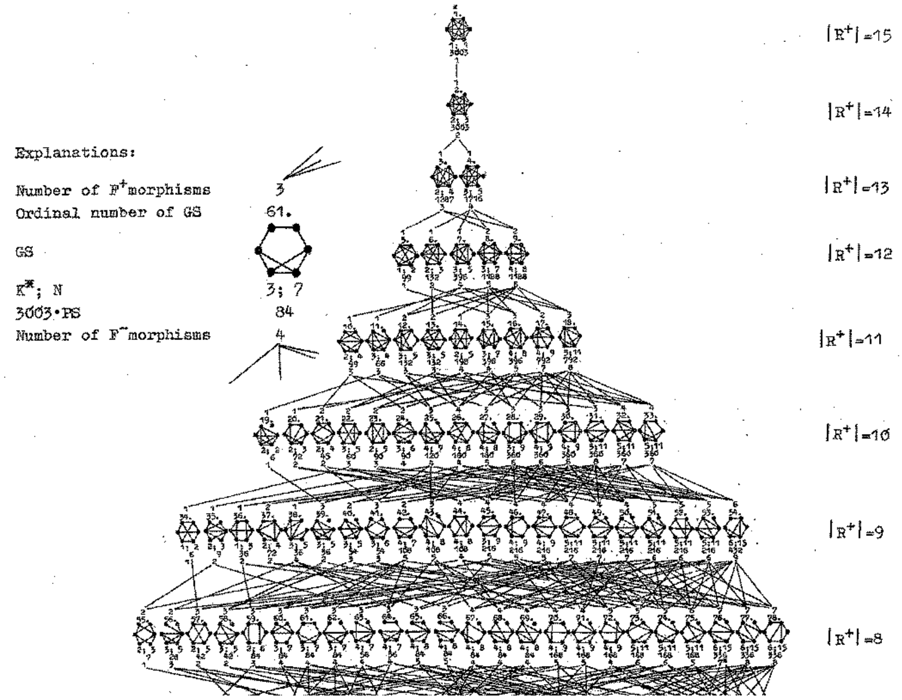
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	***i***	***k***
0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	*B*	*G2*	*H1*	*F3*	*H2*	*F2*	*H3*	***J***	**1**	*1*
	0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	*G1*	*B*	*F1*	*H1*	***I***	*H2*	*E*	*H3*	**2**	*1*
		0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	*B*	*G2*	*H1*	*F3*	*H2*	*F2*	**3**	*1*
			0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	*G1*	*B*	*F1*	*H1*	***I***	*H2*	**4**	*1*
				0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	*B*	*G2*	*H1*	*F3*	**5**	*1*
					0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	*G1*	*B*	*F1*	*H1*	**6**	*1*
						0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	*B*	*G2*	**7**	*1*
							0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	*G1*	*B*	**8**	*1*
								0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	**9**	*1*
									0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	**10**	*1*
										0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	**11**	*1*
											0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	**12**	*1*
												0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	*B*	*G1*	*D*	*G3*	**13**	*1*
													0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	**14**	*1*
														0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	*B*	*G1*	**15**	*1*
															0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	**16**	*1*
																0	***K***	*H3*	*E*	*H2*	***I***	*H1*	*F1*	**17**	*1*
																	0	***J***	*H3*	*F2*	*H2*	*F3*	*H1*	**18**	*1*
																		0	***K***	*H3*	*E*	*H2*	***I****i*	**19**	*1*
																			0	***J***	*H3*	*F2*	*H2*	**20**	*1*
																				0	***K***	*H3*	*E*	**21**	*1*
																					0	***J***	*H3*	**22**	*1*
																						0	***K***	**23**	*1*
																							0	**24**	*1*

Comments: **a)** The eleven initial pair signs are by sign graph $G_{p=-F}$ adjusted with exactness up to all the 18 *pair positions (orbits).* **b)** For structure recognition of transitive graphs is sufficient to represent only the first rows of sign matrix. **c)** There exist also other adjustment modes, which give the same result.

Structural equivalence is isomorphism on the aspect of pair positions (Chapter **3.2**). It is simply ascertainable by comparison (comparing) the semiotic models (Example **1**, Chapter **3.3**).

The *Ulam's Conjecture* is treated on the structural aspect (Chapter **4.2**). For each pair position (orbit) correspond also an ***adjacent structure*** (Chapter **4.3**)**.** An adjacent structure is obtained by removal or adding an edge of an orbit. The successions of adjacent structures form the ***Constructive System of Structures*** or *structural reconstructions* denoted by $\boldsymbol{CSR}^{|V|}$ where **|*V*|** is its *degree* (Chapter **4.4**).

Example 4. The first half of the lattice of Constructive System of Structures $\boldsymbol{CSR}^{|V=6|}$ with $|V|=6$ elements:



Comments: **a)** Each structure in this system is an adjacent structure. **b)** Each structure is ***decomposable*** to its adjacent sub-structures, as well as to its adjacent super-structures. **c)** Each structure has been ***reconstructed*** (reconstructable) by its adjacent sub-structures, as well as by its adjacent super-structures. **d)** In first example showed structure has here ordering number 22. **e)** The complements of structures located symmetrically in the second half of this system.

Summary

Structure (Latin word *structura – (inner)building*) is generally defined as a *connection-, (usually permanent) relationship or a manner of organization* of system's elements. The usage of the term "structure" has evolved over time in different languages and societies where it refers to different and sometimes vague concepts. Structure semiotics assign to the concept of "structure" its inherent (intrinsic, specific) meaning and content: Structure is the ***complete invariant of isomorphic graphs***, i. e. a system of invariant attributes of structure**.** It is represented canonically as a *text* of structure in the form of its *semiotic model* **SM**.

Here deal with a rather delicate topic. Firstly, nowadays no exist an adopted for all definition of the structure, secondly, some mathematicians do not accept the using of elements of semiotics, and thirdly, semioticians can not understand the relations with the graphs.

Nevertheless, structure semiotics reveals the "hidden sides" of the structure and to solve some of the classic problems in non-classical manner, and to set and solve the new ones. The main problems are *complete canonical representation of the structure* and constructing theirs *system* which is associated with the *Reconstruction Problem.*

Time complexity the *recognition of structure* and *ascertainment of structural equivalence* be depend only at the number of vertex pairs. It is not comparable with isomorphism detection techniques. Structure semiotics constitutes a complex of heuristic methods for studying the structure and its properties.