So as we know and can be see on the lattice can be various (non-isomorphic) structures have some common adjacent structures. The lattice demonstrates, that edge reconstructivity is true. Non-reconstructable structures cannot exist. The full text see http://ester.nlib.ee:80/record=b2297694~S1*est .

Serrman Ashay Dharwadker works on the correct proof of Ulam's Conjecture by its verbatim representation. Blazej Podsiadlo [39] was proved this Conjecture already earlier.

5. SPECIFICATIONS

5.1. Semiotic Identification Principle

Proceed from a list ***L*** of adjacent elements the Semiotic Identification Principle establish the *pair graphs* g_{ij}, for adjacent and disadjacent element pairs their *pair signs* $w_{ij}=\pm d.n.q$ and form the *sign matrix* ***S.*** It is realized on computer.

Specification 5.1. ***Semiotic Identification Principle*** by a concrete example.

S5.1.1. OPERAND: *Complete list L of adjacent elements of a graph:*

1	**2, 3, 4**
2	**1, 3, 5, 6, 7**
3	**1, 2, 6, 7**
4	**1, 8**
5	**2, 6**
6	**2, 3, 5, 9**
7	**2, 3, 9**
8	**4, 11, 12**
9	**6, 7, 10, 11**
10	**9**
11	**8, 9**
12	**8**

Comment: In principle can be the enumeration contains spaces.

S5.1.2. ***Forming the neighborhood entry Nij of an element*** v_i. Start at v_i to find by list ***L*** its adjacents v_j, further, to find adjacents of their adjacents etc. Represent these by *neighborhood partial entries* Nij_d, which be divide by the *distance* ***d*** at initial element v_i. In partial entry Nij_{d+1} do must not repeat adjacent pairs, which are presented in preceding partial entry Nij_d, i.e. in its ***j***-row to exclude elements, which be fixed in ***i***-row of preceding partial entry:

i=	***1 1 1***	***2 2 2 2 3 3 3 4***	***5 6 6 7 8 8***	***9 9 11***
j=	***2 3 4***	***3 5 6 7 2 6 7 8***	***6 5 9 9 11 12***	***10 11 9***
	$Nij_{d=1}$	$Nij_{d=2}$	$Nij_{d=3}$	$Nij_{d=4}$
		Exclude ***1***	Exclude ***2, 3, 4***	Exclude ***5, 6, 7, 8***

Comments: **a)** Partial entry $Nij_{d=5}$ do no arise, because ***9*** and ***11*** in it be excluded. **b)** In partial entries Nij_d can be exist also repeated adjacent pairs, for example such as,

2 3 ***5 6*** ***9 11***
3 2 ($Nij_{d=2}$) ***6 5*** ($Nij_{d=3}$) ***11 9*** ($Nij_{d=4}$)

S5.1.3. ***Removing the repeated adjacent pairs.*** To remove in partial entries Nij_d the repeated pairs and preserve these, where $i<j$.

i=	*1 1 1*	*2 2 2 2 3 3 4*	*5 6 7 8 8*	*9 9*
j=	*2 3 4*	*3 5 6 7 6 7 8*	*6 9 9 11 12*	$\underline{10}$ *11*
	$Nij^*_{d=1}$	$Nij^*_{d=2}$	$Nij^*_{d=3}$	$Nij^*_{d=4}$

S5.1.4. ***Forming a pair entry Bij* for case d>1.*** To find *more distant* element ***j*** for initial ***i.*** In present case it is ***j=10***. The distance ***d*** determined with Nij^*_d, where ***j*** at first time appear. For example, the distance ***d*** to ***j****=10* is 4, but to ***j****=11* is 3. In case of many more distant elements begin at smaller:

S5.1.4a. In $Nij^*_{d=max}$ to fix only these pairs, whereof ***j***-elements represent selected furthest.

S5.1.4b. Proceed from a selected more distant ***j***-to move by partial entries at right to left back and to fix in Nij^*_{d-1} only these adjacents, whereof ***j***-elements correspond to ***i***-elements of preceding Nij^*_d.

S5.1.4c. If in partial entry is fixed mutual adjacency of ***i***-elements, then it also belong to entry. By using these conditions for all the partial entries Nij^*_d we obtain *a pair entry* ***Bij****, which characterize a pair graph g_{ij} (in present case pair graph $g_{1.10}$):

i=	*1 1*	*2 2 2 3 3*	*6 7*	*9*
j=	*2 3*	*3 6 7 6 7*	*9 9*	$\underline{10}$
	$Bij^*_{d=1}$	$Bij^*_{d=2}$	$Bij^*_{d=3}$	$Bij^*_{d=4}$

By pair entry ***Bij**** to fix:

a) list of elements of pair graph $g_{1.10}$, $B_{1.10}=\{1, 2, 3, 6, 7, 9, 10\}$,

b) pair sign $dnq_{1.10}=-4.7.10$, where *–4* is *distance* ***d****, 7 – the number of elements* ***n*** and ***10*** *– the number of adjacencies (edges)* ***q*** of pair graph (def. 4).

Comment: The lists of elements of pair graphs are operands for some other functions.

S5.1.5. ***Fixing the partial entries for cases d>1 of a pair entry Bij*.*** In the formed pair entry to fix for all its ***j***-elements on the distance ***d***>1 (in present case for ***j=9, 7, 6***) their partial pair entries. For example, obtain that $B_{1.9}=\{1, 2, 3, 6, 7, 9\}$ and $dnq_{1.9}=-3.6.9$, and $B_{1.7}=\{1, 2, 3, 7\}$ and $dnq_{1.7}=-2.4.5$ etc.

S5.1.6. ***Forming all the pair entries Bij* for cases d>1.*** Proceed from ordered neighborhoods ***Nij**** to find so far no fixed more distant ***j***-elements and to form corresponding pair entries. In present case for ***j****=**12***, after it also ***11*** and ***5*** etc. ***8*** belong to ***B(i=1,j=12)****.

S5.1.7. ***Forming a pair entry Bij* for triangular case d=1.*** If in $Nij^*_{d=2}$ be fixed *mutual adjacency* with ***j***-elements of $Nij^*_{d=1}$, then to fix for $Nij^*_{d=1}$ ***j***-elements only these. Back, in $Nij^*_{d=2}$ as well to fix only these adjacent pairs. In present case we have next pair entry ***Bij****:

i=	*1 1*	*2*
j=	*2 3*	*3*
	$Bij^*_{d=1}$	$Bij^*_{d=2}$

By pair entry ***Bij**** to fix:

a) list of elements of pair graphs $g_{1.2}$ and $g_{1.3}$, $B_{1.2}=B_{1.3}=\{1, 2, 3\}$,

b) pair sign $dnq_{1.2}=dnq_{1.3}=+2.3.3$, where +***2*** is the *collateral distance +****d*** between adjacent elements.

Comment: In present case it is 3-clique. This method works also in case of arbitrary ***n***-cliques.

S5.1.8. ***Forming a pair entry Bij* for non-triangular case d=1.*** If among adjacent elements ***j*** of ***i*** exist such that do no have mutual adjacency with others adjacent elements of I, then to remove for once this connection (edge) e_{ij}. On present example is it ***j=4.*** In such case is the graph structure modified and we must to form a new neighborhood entry ***Nij**** and by it to fix corresponding (*d*>*1*)-pair entry ***Bij****:

i=	***1 1***	***2 2 2 3 3***	***6 7***	***9***	***11***	***8***
j=	***2 3***	***3 6 7 6 7***	***9 9***	***11***	***8***	***4***
	$Bij*_{d=1}$	$Bij*_{d=2}$	$Bij*_{d=3}$	$Bij*_{d=4}$	$Bij*_{d=5}$	$Bij*_{d=6}$

By pair entry ***Bij**** to fix:

a) list of elements of pair graph $g_{1.4}$, $B_{1.4}$={***1, 2, 3, 4, 6, 7, 8, 9, 11***},

b) the number of connections (edges) equal to adjacent pairs plus one ($e_{1.4}$); thus the pair sign $dnq_{1.4}$= ***+6.9.13***.

Comment: S5.1.8 can be use also for triangular cases.

S5.1.9. ***Pair entries in the branching cases d=1.*** According to def. 4 the adjacent pairs of a branch have pair entry in the form dnq_{ij}=***+1.2.1***. For example, in the case adjacent pair ***9-10***:

a) pair graph $g_{9.10}$, $B_{9.10}$={***9, 10***},

b) pair sign $dnq_{9.10}$=***+1.2.1***. Such sign has also adjacent pair ***8-12***.

S5.1.10. ***Pair signs for disconnected elements.*** Between disconnected elements do not exist pair graphs and according to def. 4 they have pair signs dnq_{ij}= ***–0.***

S5.1.11. ***Forming all the pair entries Bij* of a graph.*** To realize the tasks S5.1.2 – S5.1.10 *for all the elements* v_i, but to *exclude the reduplications*. It mean, that in case each pair ***i-j*** to check, is the entry ***Bij**** for ***j-i*** already be formed.

S5.1.12. ***Decomposition the sign matrix.*** 1) Decompose the sign matrix ***S*** by *u*-signs lexicographically to classes S_k. 2) In the framework of S_k decompose the rows and columns by *s*-signs to complementary classes S_k. 3) Repeat (2) up to complementary decomposing no arise.

RESULTS: a) *Decomposed sign matrix **S***; b) *The lists of elements* {B_{ij}} *of pair graphs.*

Example 5.1. As result of SIP obtained all the pair signs of the presented graph:

–5.9.10=A, –4.8.9=B, –4.7.10=C, –4.5.4=D, –3.6.9=E, –3.5.6=F, –3.5.5=G,
–3.4.3=H, –2.5.7=I, –2.4.5= J, –2.4.4=K, –2.3.2=L,
+1.2.1=M, +2.3.3=N, +2.4.5=O, +2.5.7=P, +3.5.7=Q, +6.9.13=R.

11	9	2	3	6	7	4	1	10	8	12	5	*i*	*ABCDEFGHIJKL**MNOPQR***	k
0	***R***	*-G*	*-G*	*-L*	*-L*	*-L*	*-H*	*-L*	***R***	*-L*	*-H*	**11**	*000000220005**000002***	1
	0	*-K*	*-K*	***Q***	***Q***	*-H*	*-E*	***M***	*-L*	*-H*	*-L*	**9**	*000010020022**100021***	2
		0	***P***	***O***	***N***	*-L*	***N***	*-G*	*-H*	*-D*	***N***	**2**	*000100210011**031100***	3
			0	***N***	***N***	*-L*	***N***	*-G*	*-H*	*-D*	*-J*	**3**	*000100210111**030100***	4
				0	*-I*	*-F*	*-J*	*-L*	*-H*	*-D*	***N***	**6**	*000101011102**021010***	5
					0	*-F*	*-J*	*-L*	*-H*	*-D*	*-L*	**7**	*000101011103**020010***	6
						0	***R***	*-D*	***R***	*-L*	*-H*	**4**	*000102020004**000002***	7
							0	*-C*	*-L*	*-H*	*-L*	**1**	*001010020202**020001***	8
								0	*-H*	*-D*	*-H*	**10**	*001200220003**100000***	9
									0	***M***	*-B*	**8**	*010000050002**100002***	10
										0	*-A*	**12**	*100500020002**100000***	11
											0	**5**	*110000030103**020000***	12

5.2. System Forming Principle

A morphism at a structure $\boldsymbol{GS}$ to its adjacent structures $\boldsymbol{GS^{adj}_n}$, $\boldsymbol{F_n: GS \rightarrow GS^{adj}_n}$, $n \in [1, N]$, represent an *elementary change* (con. 9, def. 6), where the opposite change, reverse, $\boldsymbol{F^R_n: GS^{adj}_n \rightarrow GS}$ is treated as a *reconstruction* (con. 10). Both are related with *pair orbits* $\boldsymbol{\Omega R_n}$ (con. 4).

<u>**Specification 5.2.**</u> ***Adjacency function.*** OPERAND: a) *Complete sign matrix* $\boldsymbol{S^*}$ *of an initial structure* $\boldsymbol{GS}$; b) *Its list of adjacent vertices* $\boldsymbol{L}$. ALGORITHM: 1) Proceed at conceded orbit $\boldsymbol{\Omega R_n}$, $n \in [1, N]$, fix: a) morphism probability $\boldsymbol{PF_n}$; b) coordinates $\boldsymbol{ij}$ of the first element of $\boldsymbol{\Omega R_n}$. 2) Remove (or add) $\boldsymbol{ij}$ from (to) the list $\boldsymbol{L}$. Obtained a list $\boldsymbol{L^{adj}_{ij}}$ of an adjacent graph $\boldsymbol{G^{adj}_n}$. 3) Applied to the list $\boldsymbol{L^{adj}_{ij}}$ by turns (successively) SIP for obtaining matrix $\boldsymbol{S^{adj}_n{}^*}$ of adjacent structure $\boldsymbol{GS^{adj}_n}$. 4) By coordinates $\boldsymbol{ij}$ fix in $\boldsymbol{GS^{adj}_n}$ corresponding reverse orbit $\boldsymbol{\Omega R^R_n}$. 5) Retain the original form of the initial list $\boldsymbol{L}$. RESULT: a) *Sign matrix* $\boldsymbol{S^{adj}_n{}^*}$ *of an adjacent structure* $\boldsymbol{GS^{adj}_n}$; b) *Morphism probability* $\boldsymbol{PF_n}$; c) *Reverse orbit* $\boldsymbol{\Omega R^R_n}$.

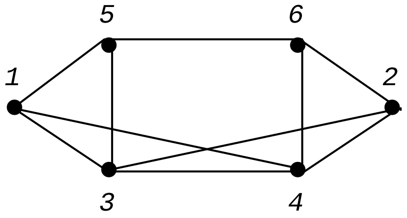
<u>Comments:</u> **a)** Metaphorically: reconstruction is not a trivial replacing a edge, but an operation with an *arbitrary* pair in the reverse orbit. **b)** Morphism is *reversible (returnable),* i.e. to each (–)morphism $\boldsymbol{F_n^-}$ correspond a (+)morphism $\boldsymbol{F_n^+}$ and contrary.

The idea of system algorithm is simple: to its operand is a *list of adjacent elements* $\boldsymbol{L^{init}}$ *only of an initiate graph*, where on the base of obtained sign matrix $\boldsymbol{W^*}$ to induce its adjacent graphs, and on the base of lasts to induce their adjacent graphs etc. To result be obtained entries for all the structures with various structural characteristics and measures and data about the system as a whole.

<u>**Specification 5.3.**</u> ***System function.*** By a concrete example.

<u>S5.3.1.</u> *Processing the graphs on the initial level* $\boldsymbol{GSR^{init}}$.

<u>**S5.3.1a.**</u> OPERAND: *List* $\boldsymbol{L^{init}}$ *of adjacent pairs of a graph* (it is graph-structure ***GS.28*** in the system $\boldsymbol{FGS^{|V|=6}}$):



```
1 – 3, 4, 5;
2 – 3, 4, 6;
3 – 1, 2, 4, 5;
4 – 1, 2, 3, 6;
5 – 1, 3, 6;
6 – 2, 4, 5;
```

<u>**S5.3.1b.**</u> Forming the decomposed sign matrix $\boldsymbol{S^*}$ by help SIP.

```
A:-2.5.7; B:-2.4.5; C:-2.4.4; D:+2.3.3; E:+2.4.5; F:+3.6.10.
```

```
 | 1   1| 2   2| 3   3|                 k
| 1   2| 3   4| 5   6|   i  ABCDEF        123
| 0   B| D   E| D   C|   1  011210   1   021
      0| E   D| C   D|   2  011210   1   021
       | 0   E| D   A|   3  100220   2   211
             0| A   D|   4  100220   2   211
              | 0   F|   5  101201   3   111
                    0|   6  101201   3   111
```

Comments: **a)** In case of the large regular graphs $|R^+|>20$ must be for structure establishment to use complementarily also algorithms 6 (conjugate graph fnc.) and/or 7 (exponentiation fnc.). **b)** If it is a single initial graph, then its existence probability $PS=1$; if the initial graphs exist more, then to decompose (partite) existence probability PS evenly; fix PS to *measure's entry*.

S5.3.1c. Identification the pair orbits ΩR_n. Beginning at partial matrix $S_{ki,kj}^*=S_{ki}^*\cap S_{kj}^*$ with *ki.kj=1.1*, then move to *1.2* etc:

1.1 *1.2* *1.3* *1.4* ...
2.2 *2.3* *2.4* ...
3.3 *3.4* ...
4.4 ...

In each partial matrix $S_{ki,kj}^*$, beginning at smallest pair sign to fix the power of pair orbit by help algorithm 3 (symmetry fnc.). To enumerate each pair orbit by common range number, $n=1,2,3,\ldots,N$, whereat the pair(-)- and (+)orbits to enumerate separately:

S5.3.1d. Measurement the structure by help measure function and fix the results to *measures entry*:

Measure's entry of graph-structure ***GS.28*:**

GS	K	N^+	N^-	MCQ	MMC	DM	HR	3003PS	SR	CPX	TRA	BRA	HE
28	3	6	3	3	4	2	3.107	360	0.205	3.210	0.900	0	2.571

Comment: The meanings of the markings see in measurement function (fnc. 8).

S5.3.2 *Forming the adjacent level GSR^{adj}.*

S5.3.2a. Inducing (forming) the adjacent structures by help adjacency function.. Each pair orbit ΩR_n corresponds to an adjacent structure GS^{adj}_n:

1) by removal a connection (edge), that belong to a pair(+)orbit, from the adjacency list, $L^{init}-ij$, obtain the adjacency list L^{low}_n of adjacent subgraph GS^{low}_n;

or/and

2) by addition a connection (edge), that belong to a pair(-)orbit, to the adjacency list, $L^{init}+ij$, obtain the adjacency list L^{upp}_n of adjacent subgraph GS^{upp}_n;

3) to determine the morphism probability PF_n and to fix it to *adjacents entry*.

S5.3.2b. Processing the adjacent structure GS^{adj}_n by its adjacency list L^{adj}_n on the ground of S5.3.1 and S5.3.2a.

S5.3.2c. With help Structural Equivalence Principle checks up, is the obtained GS^{adj}_n already in the level GSR^{adj} fixed previously:

1) If such GS^{adj}_n no fixed, then: a) assign for it current range number $m=1,2,...$ on the level GSR^{adj} and add it to corresponding entry of GS^{init}; b) to determine its preliminary existence probability $PS^{adj}=PS^{init}_n \times PF^{init}_n$; c) to retain its adjacency list L^{adj}.

2) If such GS^{adj}_n fixed, then: a) to add its range number m on the level GSR^{adj} to adjacents entry of GS^{init}; b) to determine its current existence probability PS^{adj}: $PS^{adj}+PS^{init}_n \times PF^{init}_n$.

Adjacency entry of graph structure ***GS.28*:**

GS	***n=***	***1***	***2***	***3***	***4***	***5***	***6***
GS.28	$\boldsymbol{m}^{upp}(GS^{upp}{}_{n})$ $k_i,k_j(w)$ $PF^{upp}{}_{n}$	***5****(14)* 1.1 *(-B)* *1/5*	***6****(15)* 1.3 *(-C)* *2/5*	***8****(17)* 2.3 *(-A)* 2/5	-	-	-
GS.28	$\boldsymbol{m}^{low}(GS^{low}{}_{n})$ $k_i,k_j(w)$ $PF^{low}{}_{n}$	***15****(48)* 1.2 ***(D)*** *2/10*	***19****(52)* 1.2 ***(E)*** *2/10*	***21****(54)* 1.3 ***(C)*** *2/10*	***3****(36)* 2.2 ***(E)*** *1/10*	***13****(46)* 2.3 ***(D)*** *2/10*	***12****(45)* 3.3 ***(F)*** *1/10*

Comments: **a)** There n is the range number of pair orbit ΩR_n. **b)** m is the range number of adjacent structure in upper (m^{upp}) and lower (m^{low}) adjacent level correspondingly. **c)** k_i,k_j is the coordinates of partial matrix $S_{ki,kj}$ and *(w)* corresponding pair sign.

S5.3.3. *Development the structure system* $FGS^{|V|}$.

- **S5.3.3a.** Reiteration on the framework of pair orbits ΩR_n of GS^{init}. For each $n\in[1, N]$ to use S5.3.1c, S5.3.1d and S5.3.2. If $n=N$, then go to S15.3.3b.
- **S5.3.3b.** Reiteration on the framework of structures $GS^{init}{}_{m}$. of initial level GSR^{init}. For each $m\in[1, M]$ to use S5.3.1, S5.3.2 and S5.3.3a. If $m=M$, then go to S5.3.3c.
- **S5.3.3c.** Reiteration on the framework of structural levels GSR. of the system $FGS^{|V|}$. For each $r\in[1, R]$ to use S5.3.1, S5.3.2, S5.3.3a and S5.3.3b. If $r=R$, then it is last level.
- **S5.3.3d.** RESULT: *For all the structures:* a) *Complete sign matrix* S^*; b) *Symmetry kind*; c) *Measure entries*; d) *Adjacency entries.*

5.3. Some specifications about the adjacent structures

Case 1. It is evident that adjacent structure of a *vertex symmetric structure* is *locally symmetric*.

Example 5.2. Processing results the adjacent structures of well-known bi-symmetric Peteresen graph. By *removal* at Petersen graph ***PET*** (Example 2.1) an edge i,j=4,5 is obtained its adjacent sub-structure PET^{sub}:

```
A:-4.10.14; B:-3.6.6; C:-2.3.2; D:+4.7.8; E:+4.9.12; F:+4.10.14.
```

```
| 1  1  1  1| 2  2  2  2| 3  3|               orb    s
| 2  6  7  8| 1  3  9 10| 4  5|   i   ABCDEF        123 deg
  0 -C  F -C| E  E -C -C|-C -C|   2   006021   1    120  3
     0 -C  F| E -C  E -C|-C -C|   6   006021   1    120  3
        0 -C|-C -C  E  E|-C -C|   7   006021   1    120  3
           0|-C  E -C  E|-C -C|   8   006021   1    120  3
            | 0 -C -C -C|-B  D|   1   015120   2    201  3
                 0 -C -C| D -B|   3   015120   2    201  3
                    0 -C| D -B|   9   015120   2    201  3
                       0|-B  D|  10   015120   2    201  3
                        | 0 -A|   4   124200   3    020  2
                             0|   5   124200   3    020  2
```

By *addition* to Petersen graph ***PET*** an edge i,j=4,6 is obtained its adjacent super-structure PET^{sup}:

```
A:-2.4.4; B:-2.3.2; C:+2.3.3; D:+3.4.4; E:+4.10.16.
```

```
| 1  1| 2| 3| 4  4  4  4| 5  5|            orb   s1    s2
| 2 10| 7| 9| 1  3  5  8| 4  6|  i  ABCDE       1234 12345 deg
  0 -B| E|-B| E  E -B -B|-B -B|  2  06003   1   1020 01020   3
     0| E|-B|-B -B  E  E|-B -B| 10  06003   1   1020 01020   3
        0| E|-B -B -B -B|-B -B|  7  06003   2   2100 20100   3
           0|-B -B -B -B| C  C|  9  06201   3   1002 01002   3
            | 0 -B  D -B|-A  D|  1  15021   4   1011 10011   3
                 0 -B  D| D -A|  3  15021   4   1011 10011   3
                    0 -B| D -A|  5  15021   4   1011 10011   3
                       0|-A  D|  8  15021   4   1011 10011   3
                        | 0  C|  4  23220   5   0121 00121   4
                             0|  6  23220   5   0121 00121   4
```

Common invariants and measures:

Symmetry	**|*V*|**	**|*R*|**
Local-symmetric	**10**	**45**

Distinguishing invariants and measures:

G	**|*E*|**	***K***	***N***	***SVV***	***SV***	***SRV***	***HR***	***SR***	***TRA***
PET(sub)	**14**	**3**	**9**	$2^{1}4^{2}$	*0.5419*	$1^{1}2^{1}4^{3}6^{1}8^{3}$	*0.8939*	***0.4593***	*0*
PET(sup)	**16**	**5**	**16**	$1^{2}2^{2}4^{1}$	*0.3612*	$1^{3}2^{5}4^{8}$	*1.1582*	***0.2994***	*0.188*

G	N^{+}	N^{-}	***P***	***CL***	***MC***	***DM***	SEV^{+}	***SE***
PET(sub)	3	6	6	2	5	4	$2^{1}4^{1}8^{1}$	*0.6379*
PET(sup)	7	9	5	3	5	2	$1^{2}2^{3}4^{2}$	*0.3437*

Comments: **a)** Exactly these same structures (***sub*** and ***sup***) are obtainable by operating with an arbitrary edge on Petersen graph. **b)** Adjacent sub-structure of Petersen graph has *3 vertex- and 9 pair orbits*, i.e. PET^{sub} has *9 adjacent structures*. Its adjacent super-structure has *5 vertex- and 16 pair orbits*, i.e. PET^{sup} has *16 adjacent structures* (and its symmetry value ***SR*** is smaller). **c)** From *5-girth regularity* of Petersen graph is in PET^{sub} remained 14/15 or 93%, but in PET^{sup} 7/15 or 47%. The first is „more petersenical". **d)** ***Reverse pair orbit***, that reconstruct the Petersen graph place in partial matrix $W_{3.3}$ of PET^{sub} by sign *–A*; reconstructing probability ***PF'=1/31***. ***Reverse pair orbit*** of PET^{sup} place in partial matrix $W_{5.5}$ in the form of sign ***C***; reconstructing probability ***PF'=1/16***. **e)** PET^{sub} is a common adjacent super-structure of 3 initial structures and a common adjacent sub-structure of 6 initial structures. PET^{sup} is a common adjacent super-structure of 7 initial structures and common adjacent sub-structure of 9 initial structures.

Case 2. It is evident that adjacent structure of a *strongly regular structure cannot be* strongly regular.

Example 5.3. Adjacent substructures of bisymmetric strongly regular graphs SIB_A and SIB_B (see before Example 3.5).

We know that SIB_A and SIB_B have the same symmetry properties but are *non-isomorphic*. By removal from these graphs either an edge we obtain their adjacent substructures $SIB_A\backslash e_{ij}$ (SIB_A^{sub}) and $SIB_B\backslash e_{ij}$ (SIB_B^{sub}) that have common pair signs:

-A: -2.6.8; -B: -2.5.6; -C: -2.4.5; ***+D: +2.3.3; +E: +2.4.5; +F: +2.4.6.***

SIB_A^{sub} and SIB_A^{sub} have four vertex orbits:

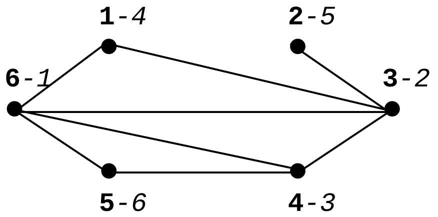
Vertex orbit ***k***	Orbit power $\lvert\Omega V_k\rvert$	Frequency vector u_i *A.B.C.**D.E.F***	Class vector s_i 1.2.3.4.
1	2	*18.9.1.**2.0. 9.***	0.9.0.2.
2	18	*26.1.0.**0.0.12.***	1.5.6.0.
3	18	*27.0.0.**0.0.12.***	0.6.5.1.
4	2	*27.0.0.**2.1. 9.***	2.0.9.1.

Comments: **a)** The substructures SIB_A^{sub} and SIB_A^{sub} must be *non-isomorphic*, so as their initial structures SIB_A and SIB_A, but have structurally *eqiuvalent usual sign matrices* S_A and S_B. **b)** For obtaining the *seep sign matrices* S^*_A and S^*_B is suitable to use the *product deep method* P3.1.3. **c)** It is suitable to use there also the Graph Isomorphism Algorithm.

Case 3. In case of the *0*-symmetric structures (i.e. all the orbits are one-elements) can be exist *separate cases* where different pair orbits $\Omega R_{n=a}$ and $\Omega R_{n=b}$ give the same adjacent structure GS^{adj}_n.

Example 5.4.. A very separate case between the adjacent structures. Graph structures GS_A *(GS.76(6.8.22))* and GS_B *(GS.100(6.7.22))* (in Graph Atlas G137 ja G113).

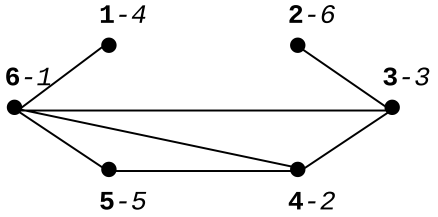
A:-3.5.6; B:-2.4.5; C:-2.3.2; ***D:+1.2.1; E:+2.3.3; F:+2.4.5.***



```
| 1| 2| 3| 4| 5| 6|                 k
| 6| 3| 4| 1| 2| 5|  i   ABCDEF    *
| 0| F| F| E|-C| E|  6   001022    1
      0| E| E| D|-B|  3   010121    2
         0|-B|-C| E|  4   011021    3
            0|-C|-C|  1   012020    4
               0|-A|  2   103100    5
                  0|  5   111020    6
```

DEG=1¹2²3¹4²

A:-3.5.6; B:-3.4.3; C:-2.4.5; D:-2.3.2; ***E:+1.2.0; F:+2.3.3; G:+2.4.5.***



```
| 1| 2| 3| 4| 5| 6|                  k
| 6| 4| 3| 1| 5| 2|  i   ABCDEFG    *
| 0| G| F| E| F|-D|  6   0001121    1
      0| F|-D| F|-D|  4   0002021    2
         0|-D|-C| E|  3   0011120    3
            0|-D|-B|  1   0103100    4
               0|-A|  5   1011020    5
                  0|  2   1102100    6
```

DEG=1²2¹3²4¹

Comments: Between graphs G_A and G_B there exist the following ***adjacency relations*:**

- Let structure GS_A is an ***initial structure***. Then structure GS_B is its ***adjacent substructure*** GS^{sub}_n of GS_A, $F: GS_A \rightarrow GS_B$. Indeed $G_A \backslash e_{1,3} \cong G_A \backslash e_{5.6} \cong G_B$. In present case the isomorphism of greatest subgraphs of graph G_A does not mean belonging of $e_{1.3}$ and $e_{5.6}$ to one and same pair orbit, $e_{1.3} \subseteq \Omega R_{(A)n=9}$ and $e_{5.6} \subseteq \Omega R_{(A)n=11}$ but induce *isomorphic greatest subgraphs* $G^{sub}_{n+} = G_B$ or *one and the same adjacent structure* GS^{adj}_n.
- Let structure GS_B is an ***initial structure***. Then structure GS_A is its ***adjacent superstructure*** GS^{sup}_n of GS_B, $F: GS_B \rightarrow GS_A$. Indeed, $G_B \cup e_{1.3} \cong G_B \cup e_{2.4} \cong G_A$, but correspondingly to sign matrix W_B^* $e_{1.3} \subseteq \Omega R_{(B)n=4}$ and $e_{2.4} \subseteq \Omega R_{(B)n=7}$. At the same time there are induced *isomorphic smallest super graphs* $G^{high}_n = G_A$ or *one and the same adjacent structure* GS^{adj}_n.

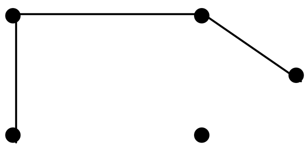
Such separate cases *do not invalidate* constructive treatment of reconstruction problem.

<u>**Case 4.**</u> An overturn the Ulam's conjecture.

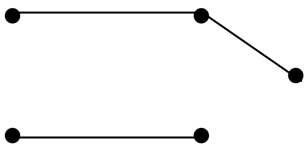
On an usual S.E.R.R. seminar in 24th April 2000. had Ants Tauts overturn the edge variant of this Conjecture by simple contra examples ***G4-2a, G4-2b*** and ***G4-1***. Later succeed it expand with examples ***G4-3a*** ja ***G4-3b.*** These contra examples were published in a S.E.R.R. issue in year 2004.

<u>**Example 5.5.**</u> A contra example the edge variant of Ulam's Conjecture on the ground of graphs ***G5-1, G5-2a, G5-2b, G5-3a*** and ***G5-3b***:

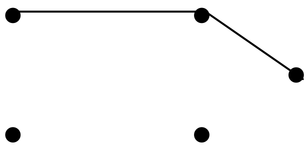
G5-3a



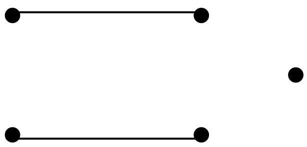
G5-3b



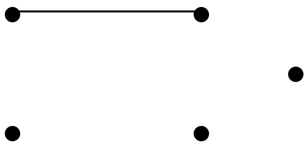
G5-2a



G5-2b



G5-1



<u>Comments:</u>

- **a)** There exist the same ***common*** ($G\backslash e_{ij}$)-sub-graphs ***G5-2a*** and ***G5-2b*** of ***non-isomorphic*** graphs ***G5-3a*** and ***G5-3b***.
- **b)** There exist the same ***common*** ($G\backslash e_{ij}$)-sub-graphs ***G5-1*** of ***non-isomorphic*** graphs ***G5-2a*** and ***G5-2b.***

Are with these contra examples Ulam's Conjecture overturn, or not? In the first case ***not***, because: 1) For changing graph ***G5-3a*** to an isomorphic with graph ***G5-2a*** exist *two* chances, but graph ***G5-3b*** has for this only *one* chance. 2) For changing graph ***G5-3a*** to an isomorphic with ***G5-2b*** exist only *one* chance, but graph ***G5-3b*** has for this *two* chances. In the second case ***yes***, because graphs ***G5-2a*** and ***G5-2b*** both have just *two* chances to changing these to isomorphic with graph ***G5-1.***

<u>**A try for rehabilitation the Conjecture.**</u> Let graph ***G*** has ***p*** vertices v_i and ***H*** has ***p*** vertices u_i, with $p>4$. If for each edge e_{ij}, the sub-graphs $G_{ij}=G\backslash e_{ij}$ and $H_{ij}=H\backslash e_{ij}$ are isomorphic, then the graphs ***G*** and ***H*** are isomorphic.

<u>Comments:</u> **a)** Ulam's Conjecture had overturns by the second contra example. Seem that this contra example be valid only then, if graphs have just four vertices. **b)** Replace the usual condition $p\geq 3$ with a new $p>4$. **c)** It no has thought, because no connected vertices can be exist many and the second case yet overturns the Conjecture.