```
<6A> <138> <135> <0m0.080s>
49538603572805247777740094585549571942722126801483633069430609557497997929616569
5336056
<6B> <137> <135> <0m0.086s>
49538603572805247777740094585549571942722126801483633069430609557497997929616569
5336056
```

Result: **Isomorphic.**

It ensues on the complete coincidence of sums or **100%** similar**.** It is a performance with two **Ramsey's graphs** that have the same structure but that are labeled differently.

As we see, the "length" of value depends on the vertex number and coincidence on relation the "lengths" of intersection and full value. It can be treat as an "isomorphism measure". Naturally, their essence needs to research.

4. THE PROBLEMS OF ADJACENT STRUCTURES AND RECONSTRUCTIONS

To adjacent structures we call the greatest sub-graphs and smallest super-graphs of a graph that are related with reconstruction problem. The reconstruction problem is well-know as the *Ulam's Conjecture* [7].

4.1. Relationships between isomorphic graphs and their $(G\backslash v_i)$-sub-graphs

Here is suitable to begin with a theorem, proved by A. Titov in 1975 [35].

Titov theorem If all the $(G\backslash v_i)$-sub-graphs of graph G are isomorphic, then automorphism group $AutG$ is transitive on the set of vertices V.

Comment: It mean that graph G *is vertex symmetric ("transitive"),* i.e. *there exists only one vertex orbit* $\Omega V_{k=1=K}$ which correspond just to one isomorphism class $\Gamma_{k=1=K}$ of $(G\backslash v_i)$-sub-graphs.

There we return anew to problems of the relationships between *equal positions* and *remaining graphs* (Prop. 2.1) and relationships between *automorphisms, local isomorphisms, orbits* and *semiotic invariants* (Prop. 2.2).

Indeed, vertex orbit $\Omega V_{k=1=K}=\Omega(v_{i=1},\ldots,v_{i=|V|})_{k=1=K}$ is *a transitivity domain of automorphisms in* $AutG$ that be expressed by an isomorphism class $\Gamma_{k=1=K}$ of $(G\backslash v_i)$-sub-graphs and is represented in the form of an *equivalence class* of vertices in partial matrix S_k of decomposed sign matrix S.

Example 4.1. Sign matrix S_A of the graph G_A (see also Example 2.9):

A:-2.8.21; B:-2.7.16;
C:+2.4.5; D:+2.5.7

```
  1   2   3   6   7   8|  4   5|  i  ABCD Orb 12
  0   B   B   C   C   C|  D   D|  1  0232  1  32
      0   B   C   C   C|  D   D|  2  0232  1  32
          0   C   C   C|  D   D|  3  0232  1  32
              0   B   B|  D   D|  6  0232  1  32
                  0   B|  D   D|  7  0232  1  32
                      0|  D   D|  8  0232  1  32
                       |  0   A|  4  1006  2  60
                              0|  5  1006  2  60
```

Comment: The orbits are recognized: graph $\boldsymbol{G_A}$ has two vertex orbits (1,2,3,6,7,8) and (4,5).

Between a vertex $\boldsymbol{\Omega V_k}$ orbit and ($\boldsymbol{G}\backslash\boldsymbol{v_i}$)-sub-graphs be valid following proposition.

Proposition 4.1. For each vertex orbit $\boldsymbol{\Omega V_k=\Omega(v_{i1},\ldots,v_{iq})_k}$, $k\in[1, K]$, correspond *an isomorphism class* $\boldsymbol{\Gamma_k}=\{(\boldsymbol{G}\backslash\boldsymbol{v_i})_1\cong\ldots\cong(\boldsymbol{G}\backslash\boldsymbol{v_i})_q\}_k$.

Comment: We state that an isomorphism class $\boldsymbol{\Gamma_k}$ is practically an “isomorphism clique”. i.e. all the pairs of ($\boldsymbol{G}\backslash\boldsymbol{v_i}$)-sub-graphs are isomorphic.

Sign matrices $\boldsymbol{S_A}$ and $\boldsymbol{S_B}$ called *structurally equivalent* if their pair signs, frequency- and class vectors coincides.

If sign matrices $\boldsymbol{S_A}$ and $\boldsymbol{S_B}$ are structurally equivalent, $\boldsymbol{S_A\approx S_B}$, then according to Prop. 2.2 are corresponding graphs $\boldsymbol{G_A}$ and $\boldsymbol{G_B}$ as a rule *isomorphic*, $\boldsymbol{G_A\cong G_B}$.

Example 4.2. Sign matrix $\boldsymbol{S_{A(i=1)}}$ of sub-graph $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=1}}$ and $\boldsymbol{S_{A(i=2)}}$ of sub-graph $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=2}}$ that are induced by the first vertex orbit of $\boldsymbol{G_A}$:

```
A:-2.7.16; B:-2.6.12; C:-0.2.0;
D:+2.4.5; E:+2.5.7.
```

```
 1|  6   7   8|  2   3   4   5|  i  ABCDE k 123    2|  6   7   8|  1   3   4   5|  i  ABCDE k 123
 0|  C   C   C|  C   C   C   C|  1  00700 1 000    0|  C   C   C|  C   C   C   C|  2  00700 1 000
  |  0   B   B|  D   D   D   D|  6  02140 2 004     |  0   B   B|  D   D   D   D|  6  02140 2 004
         0   B|  D   D   D   D|  7  02140 2 004            0   B|  D   D   D   D|  7  02140 2 004
             0|  D   D   D   D|  8  02140 2 004                0|  D   D   D   D|  8  02140 2 004
              |  0   A   E   E|  2  10132 3 032                 |  0   A   E   E|  1  10132 3 032
                     0   E   E|  3  10132 3 032                        0   E   E|  3  10132 3 032
                         0   A|  4  10132 3 032                            0   A|  4  10132 3 032
                             0|  5  10132 3 032                                0|  5  10132 3 032
```

Comments: **a)** From structural equivalence the sign matrices conclude that ($\boldsymbol{G}\backslash\boldsymbol{v_i}$)-sub-graphs $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=1}}$ and $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=2}}$ (that are induced by the first vertex orbit of $\boldsymbol{G_A}$) are *isomorphic*, i.e. these belong to the first *isomorphism class* $\boldsymbol{\Gamma_{k=1}}$. To this class belong also $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=3}}$, $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=6}}$, $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=7}}$ and $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=8}}$.

Example 4.3. Sign matrix $\boldsymbol{S_{A(i=4)}}$ of sub-graph $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=4}}$ and $\boldsymbol{S_{A(i=5)}}$ of sub-graph $\boldsymbol{G_A}\backslash\boldsymbol{v_{i=5}}$ that are induced by the second vertex orbit of $\boldsymbol{G_A}$:

```
A:-2.6.11; B:-0.2.0;
C:+2.3.3; D:+2.5.7.
```

```
5|  4|  1   2   3   6   7   8|  i  ABCD k 123  4|  5|  1   2   3   6   7   8|  i  ABCD k 123
0|  B|  D   D   D   D   D   D|  5  0106 1 006  0|  B|  D   D   D   D   D   D|  4  0106 1 006
    0|  B   B   B   B   B   B|  4  0700 2 000      0|  B   B   B   B   B   B|  5  0700 2 000
     |  0   A   A   C   C   C|  1  2131 3 103       |  0   A   A   C   C   C|  1  2131 3 103
            0   A   C   C   C|  2  2131 3 103              0   A   C   C   C|  2  2131 3 103
                0   C   C   C|  3  2131 3 103                  0   C   C   C|  3  2131 3 103
```

```
0  A  A| 6  2131 3 103          0  A  A| 6  2131 3 103
   0  A| 7  2131 3 103             0  A| 7  2131 3 103
      0| 8  2131 3 103                0| 8  2131 3 103
```

Comments: **a)** From structural equivalence the sign matrices conclude that ($G\backslash v_i$)-sub-graphs $G_A\backslash v_{i=4}$ and $G_A\backslash v_{i=5}$ (that are induced by the second vertex orbit of G_A) are *isomorphic*, i.e. these form the second *isomorphism class* $\Gamma_{k=2}$. **b)** From no-equivalence the sign matrices (Examples 4.2 and 4.3) conclude that ($G\backslash v_i$)-sub-graphs of the first isomorphism class $\Gamma_{k=1}$ no are isomorphic with the ($G\backslash v_i$)-sub-graphs of the second isomorphism class $\Gamma_{k=2}$.

Corollary 4.1. So it is demonstrated that for each vertex orbit ΩV_k, $k\in[1, K]$, correspond *an isomorphism class* Γ_k of ($G\backslash v_i$)-sub-graphs.

Example 4.5. Sign matrix S_B of the graph G_B (see also Example 2.9):

```
          A:-2.8.21; B:-2.7.16;
           C:+2.4.5; D:+2.5.7

 2  3  4  5  6  7| 1  8|  i  ABCD Orb 12
 0  C  B  B  C  C| D  D|  2  0232  1  32
    0  C  C  B  B| D  D|  3  0232  1  32
       0  B  C  C| D  D|  4  0232  1  32
          0  C  C| D  D|  5  0232  1  32
             0  B| D  D|  6  0232  1  32
                0| D  D|  7  0232  1  32
                 | 0  A|  1  1006  2  60
                      0|  8  1006  2  60
```

Comment: From structural equivalence the sign matrices S_A and S_B conclude isomorphism $G_A\cong G_B$.

Isomorphism recognition on the level of substitutions does not recognize the orbits, but the substitutions stay after all to the frameworks of orbits.

According to Prop. 3.5 accompany with the isomorphism of graphs G_A and G_B the *isomorphism of orbits*, $(\Omega V_k)_A\cong(\Omega V_k)_B$, $k\in[1, K]$, of these graphs.

NB! An orbit ΩV_k can be contains also only one element, such case called *trivial orbit*.

From isomorphism of orbits ensue according to Prop. 4.1 next proposition.

Proposition 4.2. From orbit isomorphism, $(\Omega V_k)_A\cong(\Omega V_k)_B$, $k\in[1, K]$, of isomorphic graphs G_A and G_B ensue the *isomorphism of isomorphism classes*, $(\Gamma_k)_A\cong(\Gamma_k)_B$, $k\in[1, K]$, where $(G\backslash v_i)_A\subset(\Gamma_k)_A$ and $(G\backslash v_i)_B\subset(\Gamma_k)_B$.

Comment: Isomorphism of isomorphism classes, $(\Gamma_k)_A\cong(\Gamma_k)_B$, of ($G\backslash v_i$)-sub-graphs, that accompany with isomorphism of vertex orbits, $(\Omega V_k)_A\cong(\Omega V_k)_B$, constitute practically *an union of isomorphism classes* $(\Gamma_k)_A\cup(\Gamma_k)_B$, where exist also inter-class isomorphisms $(G\backslash v_i)_A\cong(G\backslash v_i)_B$,

Look the ($G\backslash v_i$)-sub-graphs of graph G_B (Example 4.2) that is isomorphic with G_A?

Example 4.5. The sign matrix $S_{B(i=3)}$ of sub-graph $G_B\backslash v_{i=3}$ and sign matrix $S_{B(i=6)}$ of $G_B\backslash v_{i=6}$ of the first vertex orbit of G_B:

```
A:-2.7.16; B:-2.6.12; C:-0.2.0;
     D:+2.4.5; E:+2.5.7.
```

```
 3| 2  4  5| 1  6  7  8| i  ABCDE k 123     6| 2  4  5| 1  3  7  8| i  ABCDE k 123
 0| C  C  C| C  C  C  C| 3  00700 1 000     0| C  C  C| C  C  C  C| 6  00700 1 000
  | 0  B  B| D  D  D  D| 2  02140 2 004      | 0  B  B| D  D  D  D| 2  02140 2 004
       0  B| D  D  D  D| 4  02140 2 004           0  B| D  D  D  D| 4  02140 2 004
          0| D  D  D  D| 5  02140 2 004              0| D  D  D  D| 5  02140 2 004
           | 0  E  E  A| 1  10132 3 032               | 0  E  E  A| 1  10132 3 032
                0  A  E| 6  10132 3 032                    0  A  E| 3  10132 3 032
                   0  E| 7  10132 3 032                       0  E| 7  10132 3 032
                      0| 8  10132 3 032                          0| 8  10132 3 032
```

Comments: **a)** Exactly the same state that in case of first vertex orbit of G_A, but distribution differ **b)** Thus, to the *first isomorphism class* $\Gamma_{k=1}$ belong $G_B\backslash v_{i=2}$, $G_B\backslash v_{i=3}$, $G_B\backslash v_{i=4}$, $G_B\backslash v_{i=5}$ $G_B\backslash v_{i=6}$ and $G_B\backslash v_{i=7}$. **c)** From isomorphism $G_A \cong G_B$ conclude *inner-classes isomorphisms*: $(G\backslash v_{i=1})_A \cong (G\backslash v_{i=2})_B$, $(G\backslash v_{i=2})_A \cong (G\backslash v_{i=3})_B$, $(G\backslash v_{i=3})_A \cong (G\backslash v_{i=4})_B$, $(G\backslash v_{i=6})_A \cong (G\backslash v_{i=5})_B$, $(G\backslash v_{i=7})_A \cong (G\backslash v_{i=6})_B$, $(G\backslash v_{i=8})_A \cong (G\backslash v_{i=7})_B$.

Corollary 4.2. So it is demonstrated that if graphs G_A and G_B are isomorphic then in the case of each isomorphic isomorphism class, $(\Gamma_k)_A \cong (\ \Gamma_k)_B$, exist the *inter-classes isomorphisms* $(G\backslash v)_{kA} \cong (G\backslash v)_{kB}$.

According to Titov theorem a vertex symmetric graph has only one isomorphism class $\Gamma_{k=1=K}$ of $(G\backslash v_i)$-sub-graphs, i.e. all its $(G\backslash v_i)$-sub-graphs are isomorphic.

Let the $(G\backslash v_i)$-sub-graphs are results of *pair-wise disjunctive* $(G\backslash v_i)$*-operations* on the framework of a vertex orbit $\Omega V_k = \Omega(v_{i1},\dots,v_{iq})_{k=1=K}$ of graphs G_A and G_B.

Example 4.7. The sign matrices of $(G\backslash v_i)$-sub-graphs of *vertex symmetric* graphs PRA_A and PRA_B (see Example 3.3). 1) Sign matrix of $(G\backslash v_i)$-sub-graph $PRA_A\backslash v_{i=10}$ of PRA_A and 2) Sign matrix of $(G\backslash v_i)$-sub-graph $PRA_B\backslash v_{i=10}$ of PRA_B. These have common pair signs:

A:-4.15.29; B:-3.8.10; C:-3.7.8; D:-3.6.7; E:-3.6.6; F:-3.5.5; G:-3.4.3;
H:-2.4.4; I:-2.3.2;J:-0.2.0;
K:+2.3.3; L:+2.4.6; M:+3.6.9; N:+3.8.16;

```
10| 6 13| 9| 5 14| 1 17|11 12| 7  8 15 16| 3  4 19 20| 2 18| i  ABCDEFGHI JKLMN  Orb
 0| J  J| J| J  J| J  J| J  J| J  J  J  J| J  J  J  J| J  J|10  000000000190000   1
  | 0  H| C| L  N| H  I| F  F| L  L  H  H| H  H  D  D| N  D| 6  001302061 10302   2
       0| C| N  L| I  H| F  F| H  H  L  L| D  D  H  H| D  N|13  001302061 10302   2
          0| I  I| M  M| K  K| D  D  D  D| H  H  H  H| I  I| 9  002400044 12020   3
           | 0  H| N  B| D  D| L  L  H  H| H  H  D  D| H  I| 5  010400062 10302   4
                0| B  N| D  D| H  H  L  L| D  D  H  H| I  H|14  010400062 10302   4
                 | 0  I| H  H| H  H  D  D| L  L  D  D| L  E| 1  010410052 10311   5
                      0| H  H| D  D  H  H| D  D  L  L| E  L|17  010410052 10311   5
                       | 0  K| I  B  B  I| M  H  M  H| I  I|11  020202044 12020   6
                            0| B  I  I  B| H  M  H  M| I  I|12  020202044 12020   6
                             | 0  L  N  H| N  H  B  I| H  D| 7  020300062 10302   7
                                  0  H  N| H  N  I  B| H  D| 8  020300062 10302   7
                                     0  L| I  B  H  N| D  H|15  020300062 10302   7
                                        0| B  I  N  H| D  H|16  020300062 10302   7
                                         | 0  L  I  B| L  G| 3  020300152 10311   8
                                              0  B  I| L  G| 4  020300152 10311   8
                                                 0  L| G  L|19  020300152 10311   8
                                                    0| G  L|20  020300152 10311   8
                                                     | 0  A| 2  100310234 10301   9
                                                          0|18  100310234 10301   9
```

```
10| 6 13| 5 14| 1 19| 9 11|12| 7 16| 8 15| 4 20| 3 17| 2 18| i  ABCDEFGHI JKLMN Orb
 0| J  J| J  J| J  J| J  J| J| J  J| J  J| J  J| J  J| J  J|10  000000000190000  1
  | 0  H| L  N| H  I| F  C| F| L  H| L  H| H  D| H  D| N  D| 6  001302061 10302  2
       0| N  L| I  H| C  F| F| H  L| H  L| D  H| D  H| D  N|13  001302061 10302  2
        | 0  H| N  D| I  D| D| L  H| L  H| H  D| H  B| H  I| 5  010400062 10302  3
             0| D  N| D  I| D| H  L| H  L| D  H| B  H| I  H|14  010400062 10302  3
```

```
| 0  D| M  H| H| H  B| H  D| L  D| L  I| L  E| 1  010410052 10311  4
     0| H  M| H| B  H| D  H| D  L| I  L| E  L|19  010410052 10311  4
      | 0  K| K| D  I| B  D| H  H| H  M| I  I| 9  011301044 12020  5
           0| K| I  D| D  B| H  H| M  H| I  I|11  011301044 12020  5
              0| B  B| I  I| M  M| H  H| I  I|12  020202044 12020  6
               | 0  H| L  N| H  I| N  D| H  D| 7  020300062 10302  7
                    0| N  L| I  H| D  N| D  H|16  020300062 10302  7
                     | 0  H| N  B| H  I| H  D| 8  020300062 10302  8
                          0| B  N| I  H| D  H|15  020300062 10302  8
                           | 0  I| L  B| L  G| 4  020300152 10311  9
                                0| B  L| G  L|20  020300152 10311  9
                                 | 0  D| L  G| 3  020300152 10311 10
                                      0| G  L|17  020300152 10311 10
                                       | 0  A| 2  100310234 10301 11
                                            0|18  100310234 10301 11
```

Comments: **a)** ($G\backslash v_i$)-sub-graphs of poly-symmetric graphs are *partially symmetric*. **b)** The sub-graphs $PRA_A\backslash v_{i=10}$ of PRA_A and $PRA_B\backslash v_{i=10}$ of PRA_B have 14 common pair signs but their sign matrices no are equivalent. Thus $PRA_A\backslash v_{i=10}$ of PRA_A and $PRA_B\backslash v_{i=10}$ of PRA_B are *non-isomorphic.* **c)** From vertex symmetry of PRA_A and PRA_B conclude that all the ($PRA_A\backslash v_i$)-sub-graphs form an *isomorphism class* $\Gamma_{A(k=1=K)}$ and all the ($PRA_B\backslash v_i$)-sub-graphs form also an *isomorphism class* $\Gamma_{B(k=1=K)}$.

Corollary 4.3. From non-isomorphism of sub-graphs $PRA_A\backslash v_{i=10}$ and $PRA_B\backslash v_{i=10}$ conclude *non-isomorphism* of graphs PRA_A and PRA_B.

4.2. The adjacent structures: greatest sub- and smallest superstructures

There exist elementary operations with edges: 1) *removal an edge* $G\backslash e_{ij}$ give a ***greatest sub-graph*** G^{sub} of G; 2) *addition an edge* $G\cup e_{ij}$ give a ***smallest super-graph*** G^{sup} of G.

Definition 4.1. The greatest sub-graphs G^{sub} and smallest super-graphs G^{sup} both called ***adjacent graphs*** G^{adj} of graph G.

Proposition 4.3. If the edge operations f have been applied *disjunctively*, $\{(f_{ij})_1\vee\ldots\vee(f_{ij})_q\}_n$ to the vertex pairs of an pair orbit $\Omega R_n=\Omega(r_{ij1},\ldots,r_{ijq})_n$ of a graph G, then the disjunctive adjacent graphs of graph G form *an isomorphism class* $\Gamma_n=\{(G^{adj}{}_n)_1\cong\ldots\cong(G^{adj}{}_n)_q\}$.

Comment: All the graphs G^{adj} of an isomorphism class Γ_n have the same structure and represent an ***adjacent structure*** GS^{adj} of G is expressed by corresponding *sign matrix* S^{adj}.

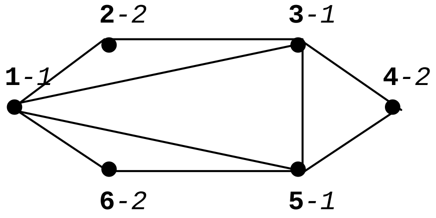
For differentiate are the serial numbers of pair(–)orbits (i.e. “non-edge” orbits) denoted by $n^{-}\in\{1,\ldots,N^{-}\}$ and pair(+)orbits (i.e. edge orbits) $n^{+}\in\{1,\ldots,N^{+}\}$, where $N^{-}+N^{+}=N$.

Definition 4.2. An edge operation that disjunctively transforms structure GS to an adjacent structure $GS^{adj}{}_n$ is called ***morphism*** F_n, $F_n\colon GS\rightarrow GS^{adj}{}_n$.

Therefore, morphism $F_{n-}\colon GS\rightarrow GS^{sup}{}_{n-}$, that is applied to a *pair*(–)*orbit* ΩR_{n-} of GS induce an *adjacent super-structure* $GS^{sup}{}_{n-}$ of GS and morphism $F_{n+}\colon GS\rightarrow GS^{sub}{}_{n+}$ that is applied to a *pair*(+)*orbit* ΩR_{n+} of GS induce an *adjacent sub-structure* $GS^{sub}{}_{n+}$ of GS.

Example 4.7. Graph structure ***GS.37 (6.9.4)*** (by Graph Atlas [36] G163), its *adjacent super- and sub-structures* and Entry of identifiers of adjacent structures and characteristics of morphisms $\boldsymbol{F_n}$:
:

```
A:-2.4.5; B:-2.3.2; C:+2.3.3; D:+2.4.5.
```

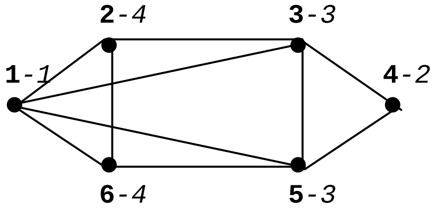


```
| 1  1  1| 2  2  2|              k
| 1  3  5| 2  4  6|  i   ABCD         12
| 0  D  D| C –A  C|  1   1022    1   22
      0  D| C  C -A|  3   1022    1   22
         0|-A  C  C|  5   1022    1   22
          | 0 -B -B|  2   1220    2   20
               0 -B|  4   1220    2   20
                  0|  6   1220    2   20
```

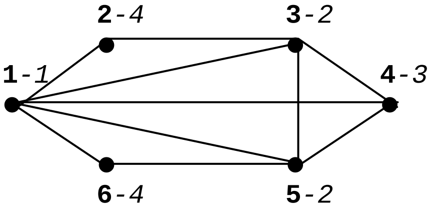
GS	GS^{adj}_n	1	2
GS.37	GS^{sup}_{n-} k.k’(p) PF^{sup}_{n-}	29 2.2 (-B) 3/6	30 1.2 (-A) 3/6
GS.37	GS^{sub}_{n+} k.k’(p) PF^{sub}_{n+}	72 1.1 (+D) 3/9	76 1.2 (+C) 6/9

Where, $\boldsymbol{GS^{sup}_{n-}}$ and $\boldsymbol{GS^{sub}_{n+}}$ – range number adjacent structure correspondingly; ***k,k’*** – index of partial matrix $\boldsymbol{S_{k,k'}}$ that contain binary orbit, where ***(p)*** to concretize the pair orbit in therein; $\boldsymbol{PF_n}$ – percentage of pair orbit or *morphism probability.*

Adjacent super-structures by pair orbit *–B*, $\boldsymbol{GS^{sup}_{n=-B}}$, (***GS.29*** *(6.10.11),* by Graph Atlas G184) and by pair orbit *–A*, $\boldsymbol{GS^{sup}_{n=-A}}$, (***GS.30*** *(6.10.12),* by Graph Atlas G180) and their matrices ***S***:



DEG=2¹3²4³



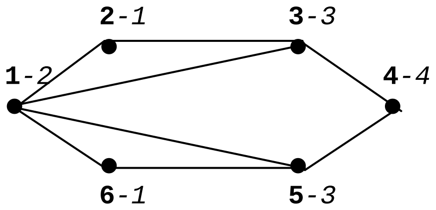
DEG=2²3¹4²5¹

```
A:-2.5.8; B:-2.4.5; C:-2.3.2;
D:+2.3.3; E:+2.4.5.

| 1| 2| 3  3| 4  4|             k
| 1| 4| 3  5| 2  6| i ABCDE     1234
| 0|-B| E  E| E  E| 1 01004 1 0022
   | 0| D  D|-C -C| 4 01220 2 0020
      | 0  E| D -A| 3 10022 3 1111
           0|-A  D| 5 10022 3 1111
            | 0  D| 2 10121 4 1011
                 0| 6 10121 4 1011
```

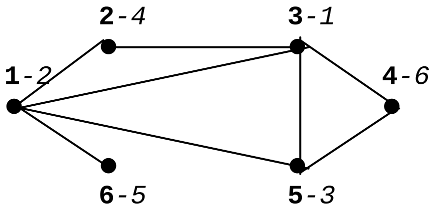
```
A:-2.4.5; B:-2.3.2;
C:+2.3.3; D:+2.4.6; E:+2.5.8.

| 1| 2  2| 3| 4  4|             k
| 1| 3  5| 4| 2  6| i ABCDE     1234
| 0| E  E| D| C  C| 1 00212 1 0212
   | 0  D| D| C -A| 3 10121 2 1111
         0| D|-A  C| 5 10121 2 1111
          | 0|-A -A| 4 20030 3 1200
             | 0 -B| 2 21200 4 1100
                  0| 6 21200 4 1100
```

Comments: All the adjacent super-graphs by pair orbit *–B*, $\boldsymbol{GS^{sup}_{n=-B}}$, are *isomorphic* and all the adjacent super-graphs by pair orbit *–A*, $\boldsymbol{GS^{sup}_{n=-A}}$, are also *isomorphic.*

Adjacent sub-structures by pair orbit +***D***, $\boldsymbol{GS^{sub}_{n=+D}}$, (***GS.72*** *(6.8.18),* by Graph Atlas G148) and by pair orbit +***C***, $\boldsymbol{GS^{sub}_{n=+C}}$, (***GS.76*** *(6.8.22),* by Graph Atlas G137) and their matrices ***S***:



DEG=$2^1 3^2 4^3$



DEG=$2^2 3^1 4^2 5^1$

A:-2.4.4; B:-2.3.2;
C:+2.3.3; D:+3.4.4.

A:-3.5.6; B:-2.4.5; C:-2.3.2;
D:+1.2.1; E:+2.3.3; F:+2.4.5.

```
| 1   1| 2| 3   3| 4|          k
| 2   6| 1| 3   5| 4| i ABCD      1234
| 0  -B| C| C  -B|-B| 2 0320 1    0110
      0| C|-B   C|-B| 6 0320 1    0110
       | 0| C   C|-A| 1 1040 2    2020
          | 0  -A| D| 3 1121 3    1101
                0| D| 5 1121 3    1101
                 | 0| 4 1202 4    0020
```

```
| 1| 2| 3| 4| 5| 6|             k
| 3| 1| 5| 2| 6| 4| i   ABCDEF     123456
  0| F| F| E|-C| E| 3   001022 1 011101
   | 0| E| E| D|-B| 1   010121 2 101110
      | 0|-B|-C| E| 5   011021 3 110001
         | 0|-C|-C| 2   012020 4 110000
            | 0|-A| 6   103100 5 010000
               | 0| 4   111020 6 101000
```

<u>Comment:</u> All the adjacent sub-graphs by pair orbit +***D***, $\boldsymbol{GS^{sub}_{n=+D}}$, are *isomorphic* and all the adjacent sub-graphs by pair orbit +***C***, $\boldsymbol{GS^{sub}_{n=+C}}$, are also *isomorphic.*

<u>**Corollary 4.4.**</u> Let there exist a isomorphic structures $\boldsymbol{G_A}$ and $\boldsymbol{G_B}$**.** Then, in the case of each pair of pair orbits $(\boldsymbol{\Omega R_n})_A$ and $(\boldsymbol{\Omega R_n})_B$, $n\in[1, N]$, according to Prop. 4.2 exist isomorphic isomorphism classes, $(\boldsymbol{\Gamma_n})_A \cong (\boldsymbol{\Gamma_n})_B$, $n\in[1, N]$, of corresponding adjacent graphs $(\boldsymbol{G^{adj}_n})_A$ and $(\boldsymbol{G^{adj}_n})_B$, where $(\boldsymbol{G^{adj}})_A \subset (\boldsymbol{\Gamma_n})_A$ and $(\boldsymbol{G^{adj}})_B \subset (\boldsymbol{\Gamma_n})_B$.

<u>Comment:</u> We already know, that isomorphic graphs $\boldsymbol{G_A \cong G_B \cong G_C} \cong \ldots$ have isomorphic adjacent graphs $(\boldsymbol{G^{adj}_n})_A \cong (\boldsymbol{G^{adj}_n})_B \cong (\boldsymbol{G^{adj}_n})_C \cong \ldots$.

Different adjacent structures $\boldsymbol{G^{adj}_n}$, $n\in[1, N]$, of a graph ***G*** are *non-isomorphic* as a rule.

<u>**Proposition 4.4.**</u> If the morphisms $\boldsymbol{F_n: GS \rightarrow GS^{adj}_n}$ have been disjunctively applied $\boldsymbol{F_1 \vee \ldots \vee F_n \vee \ldots \vee F_N}$ to the orbits $\boldsymbol{\Omega R_1, \ldots, \Omega R_n, \ldots, \Omega R_N}$ of a graph structure ***GS***, then we say that structure ***GS*** is ***deconstructed*** *to its adjacent structures* $\{\boldsymbol{GS^{adj}_n}\} = \boldsymbol{GS^{adj}_1, \ldots, GS^{adj}_n, \ldots, GS^{adj}_N}$.

<u>Comments:</u> **a)** Indivisible structures cannot exist. **b)** We already know, that adjacent structure $\boldsymbol{GS^{adj}_n}$ mean an *isomorphism class* $\boldsymbol{\Gamma^{adj}_n}$ that can be contain isomorphic adjacent graphs $(\boldsymbol{G^{adj}_n})_1 \cong (\boldsymbol{G^{adj}_n})_2 \cong (\boldsymbol{G^{adj}_n})_3 \cong \ldots \subseteq \boldsymbol{GS^{adj}_n} = \boldsymbol{\Gamma^{adj}_n}$.

4.3. On the Ulam's Conjecture

Reconstruction means anything restore. On the viewpoint of Ulam's Conjecture [7] mean it a relationship between two graphs and their $(\boldsymbol{G} \backslash v_i)$-sub-graphs. We demonstrate that the graph reconstructions are in the structure semiotics aspect a definitely solvable problem by vertex orbits $\boldsymbol{\Omega V_k}$ and corresponding isomorphism classes of $(\boldsymbol{G} \backslash v_i)$-sub-graphs. We demonstrate also that in addition to vertex reconstructions there exist also edge and "non-edge" reconstructions. It can be to construct a "constructive system of reconstructions".

The reconstruction problem is a very intensively examinee field in last half century of graph theory. The research were continue for a long time. The publications are quantities (Google are fixed 1.360.000 cases). But the ultimate solutions have only some graph classes. What for so?

We issue from this that graph G is represented in the canonical form, i.e. in the form of *complete sign matrix* S^* that represent G with exactness up to isomorphism and orbits. It mean that all the isomorphic graphs, $\{G_1 \cong \ldots \cong G_m \cong \ldots \cong G_M\}_k$ and only these, form an *isomorphism class* Γ_k that is presented by corresponding *equivalent sign matrices* $S^*_1 \approx \ldots \approx S^*_m \approx \ldots \approx S^*_M$

The classical treatment of reconstruction, the well-known *Ulam's Conjecture* is formulated as follows: "Let graph G has p vertices v_i and H has p vertices u_i, with $p \geq 3$. If for each i, the sub-graphs $G_i = G \backslash v_i$ and $H_i = H \backslash u_i$ are isomorphic, then the graphs G and H are isomorphic".

Evidently be interested on the question**:** contain the collection of sub-graphs $G \backslash v_i$ of G enough information about graph G itself? Here is suitable remark that the enough information about a graph be contained in the sign matrix S^*.

On structure semiotic aspect:

- Isomorphic graphs G and H constitute no more than *different graphs of one and the same isomorphism class* Γ, $G \& H \subset \Gamma$, that have *one and the same structure* GS and *orbits* that are presented by equivalent sign matrices $S_G^* \approx S_H^*$.
- All the isomorphic sub-graphs $G \backslash v_i$ and $H \backslash u_i$ divide to common *sub-structures* $(GS \backslash v_i)_1, \ldots, (GS \backslash v_i)_k, \ldots, (GS \backslash v_i)_K$ of isomorphic graphs G and H.

According to grand old man W.T.Tutte [37] the solution of reconstruction problem must start just from *isomorphism classes.*

Conclusion. To ignore the wording of Ulam's Conjecture, but not its sense, i.e.: **1)** To study the relationship between isomorphic graphs and their $(G \backslash v_i)$- and $(G \backslash e_{ij})$-sub-graphs. **2)** At the level of "pair-isomorphism" $G \cong H$ go to the level of *isomorphism classes* Γ, i.e. to the level of structure.

4.4. Reconstruction: an opposite operation of deconstruction

To *reconstruction* of a graph structure GS we take there reconstructability the structure by its adjacent structures GS^{adj}_n.

Proposition 4.5. If structure GS is ***deconstructed*** to its adjacent ***substructures*** $GS^{sub}_1, \ldots, GS^{sub}_n, \ldots, GS^{sub}_N$, i.e. to its greatest substructures $(GS \backslash e_{ij})_n$, then their ***union*** $\cup (GS \backslash e_{ij})_n$, $n^+ \in [1, N^+]$, ***reconstruct*** the same structure GS.

Comments:

1. Let G is represented in the form of adjacent matrix E. Deconstruction the structure GS to its adjacent substructures mean by Prop. 4.4 strictly disjunctive removal an edge $(E \backslash e_{ij})_1 \vee \ldots \vee (E \backslash e_{ij})_n \vee \ldots \vee (E \backslash e_{ij})_N$ in the case of each edge or pair(+)orbit ΩR_n. This mean that in each adjacent matrix E_n lack a connection. Consequently their union reconstruct the graph, $(E \backslash e_{ij})_1 \cup \ldots \cup (E \backslash e_{ij})_n \cup \ldots \cup (E \backslash e_{ij})_N = E, E \equiv G$.
2. Principally be valid (1) also then, when graph is presented by its list L of adjacent vertices $(L \backslash e_{ij})_1 \cup \ldots \cup (L \backslash e_{ij})_n \cup \ldots \cup (L \backslash e_{ij})_N = L, L \equiv G$.

3. If we wish to compare graphs G_A and G_B amongst then to first step is representing their in the form of sign matrices S_A and S_B. If these are structurally equivalent, $S_A \approx S_B$ then by Prop. 2.2 are the graphs isomorphic, $G_A \cong G_B$. This mean that these have *equal number N* pair orbits ΩR_n *with equal powers* $card|\Omega R_n|$, which in both matrices S_A and S_B be located in *equal positions*. Their different numbering no have matter, because we operate with unambiguous elements $(e_n)_A$ and $(e_n)_B$ of orbits $(\Omega R_n)_A$ ja $(\Omega R_n)_B$. Therefore, these have *equal number N equal adjacent structures* GS^{adj}_n, $n \in [1, N]$, i.e. isomorphic sub-graphs $(G_A \setminus e_n) \cong (G_B \setminus e_n)$ and isomorphic super-graphs $(G_A \setminus e_n) \cong (G_B \setminus e_n)$.
4. If a graph **G** that belong to isomorphism class **Γ** is *reconstructable*) by its adjacent structures GS^{adj}_n, then are so also all other graphs G_B, G_C, G_D, **…** of isomorphism class **Γ.**
5. Thus, Ulam's Conjecture is true, but its set up is *destructive* and it verbatim tracking on the ground of canonically represented, i.e. in the form of sign matrices represented graphs is *senseless*.

Proposition 4.6. If structure **GS** is ***deconstructed*** to its adjacent ***superstructures*** $GS^{sup}_1, \dots, GS^{sup}_n, \dots, GS^{sup}_N$, i.e. to its smallest superstructures $(GS \cup e_{ij})_n$, then their ***intersection*** $\bigcap (GS \cup e_{ij})_n$, $n^- \in [1, N^-]$, ***reconstruct*** the same structure **GS**.

Comment: *Intersection* be valid also in case of corresponding adjacent matrix, $\bigcap (E \cup e_{ij})_n = E$, $E \equiv G$ and lists $\bigcap (L \cup e_{ij})_n = L$, $L \equiv G$

Corollary 4.5. If structure **GS** is on the ground of its pair orbits ΩR_n, $n \in [1, N]$, *deconstructable* to its adjacent structures, $F_n: GS \rightarrow GS^{adj}_n$, then it is ***reconstructable*** both by *an union of adjacent substructures* $\bigcup GS^{sub}_n$, $n^+ \in [1, N^+]$, and by *an intersection of adjacent superstructures* $\bigcap (GS \cup e_{ij})_n$, $n^- \in [1, N^-]$.

Comments: **a)** *Edge symmetric* graph **G** has only *one* adjacent substructure $GS^{sub}_{n+=1=N+}$. If it is also *bisymmetric*, then it has also *one* adjacent superstructure $GS^{sup}_{n-=1=N-}$. **b)** By formulation the Ulam's Conjecture must be speak on two isomorphic graphs G_A and G_B that have p edges and have correspondingly p pairs of isomorphic adjacent graphs $G_A{}^{sub}{}_p \cong G_B{}^{sub}{}_p$, $p \in [1, P]$.

It is possible to set up another kind of reconstruction.

Proposition 4.7. Morphism **F** is *reversible* **–** in each adjacent structure GS^{adj} of **GS** exists an "*opposite orbit*" $\Omega R'$, whereat used opposite morphism **F'** *reconstruct* the initial structure **GS**, $F': GS^{adj} \rightarrow GS$

Comments: **a)** The reversing of morphism is valid both in the case of sub- GS^{sub}_{n+} and super-structures Gs^{sup}_{n-}. **b)** Indeed, structure **GS** can be reconstructed by each of its adjacent structure GS^{adj} separately. On the set $\{GS^{adj}_n\}$ of all the adjacents of **GS** there exists certain *set of opposite morphisms* $\{F'_n\}$, $n \in [1, N]$, such that each its disjunctive element $(F'_1: GS^{adj}_1 \rightarrow GS) \vee \dots \vee (F'_N: GS^{adj}_N \rightarrow GS)$ *reconstructs the structure* **GS** *separately.*

Corollary 4.6. With a morphism F_n accompany also a ***transition-*** or ***morphism probality*** $PF_n = card|\Omega R_n| : card|\mathbf{R}|$, of transition to adjacent structure $F: GS \rightarrow GS^{adj}_n$, where $card|\Omega R_n|$ is the power of orbit and $|\mathbf{R}|$ the number of edges or "non-edges" correspondingly**.**

Comment: In principle are all the graph structures **GS** also adjacent structures GS^{adj}_n of some structures.

It is known that non-isomorphic graphs can have some common adjacent structures. On the other hand, *deep sign matrix* S^* (Prop. 2.4) shows by orbits ΩR_n all the differences between adjacent structures GS^{adj}_n. Consequently, the Ulam's Conjecture can be in another way to formulate.

Structural interpretation the Ulam's Conjecture. Can be non-isomorphic graphs G_A and G_B have structurally equivalent deep sign matrices $S_A^* \approx S_B^*$, i.e. the same structure $GS_A \equiv GS_B$?

Trueness, that non-isomorphic graphs cannot have only common adjacent structures to demonstrate following system of the structures.

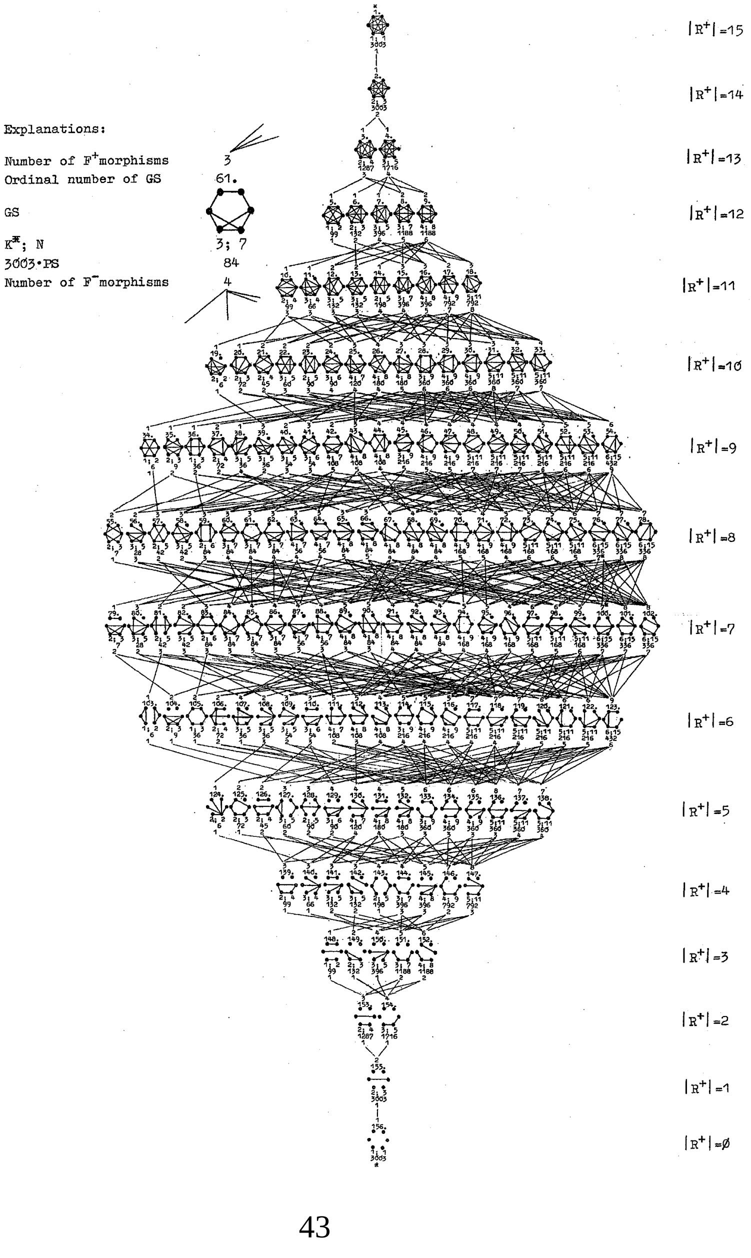
Definition 4.3. A system, that is formed from all the *structures with |V|-elements (i.e. non-isomorphic graphs with |V| vertices)* $\{GS\}^{|V|}$ and *morphisms* $\{F\}$ between these whereby:

i the set $\{GS\}^{|V|}$ is decomposed and ordered by numbers of edges **|E|** into *structural levels* ***GSL***,

ii where structures ***GS*** in ***GSL*** are ordered by essential structural measures,

we call to a ***Constructive System of Reconstructions*** or *structural changes* denoted by $CSR^{|V|}$ where **|*V*|** is its *degree* [38].

Example 4.9. The lattice of Constructive System of Recognitions $CSR^{|V=6|}$ [38] for the structures with **|*V*|** 6 elements:



So as we know and can be see on the lattice can be various (non-isomorphic) structures have some common adjacent structures. The lattice demonstrates, that edge reconstructivity is true. Non-reconstructable structures cannot exist. The full text see http://ester.nlib.ee:80/record=b2297694~S1*est .

Serrman Ashay Dharwadker works on the correct proof of Ulam's Conjecture by its verbatim representation. Blazej Podsiadlo [39] was proved this Conjecture already earlier.

5. SPECIFICATIONS

5.1. Semiotic Identification Principle

Proceed from a list ***L*** of adjacent elements the Semiotic Identification Principle establish the *pair graphs* g_{ij}, for adjacent and disadjacent element pairs their *pair signs* $w_{ij}=\pm d.n.q$ and form the *sign matrix* ***S.*** It is realized on computer.

Specification 5.1. ***Semiotic Identification Principle*** by a concrete example.

S5.1.1. OPERAND: *Complete list L of adjacent elements of a graph:*

1	**2, 3, 4**
2	**1, 3, 5, 6, 7**
3	**1, 2, 6, 7**
4	**1, 8**
5	**2, 6**
6	**2, 3, 5, 9**
7	**2, 3, 9**
8	**4, 11, 12**
9	**6, 7, 10, 11**
10	**9**
11	**8, 9**
12	**8**

Comment: In principle can be the enumeration contains spaces.

S5.1.2. ***Forming the neighborhood entry Nij of an element*** v_i. Start at v_i to find by list ***L*** its adjacents v_j, further, to find adjacents of their adjacents etc. Represent these by *neighborhood partial entries* Nij_d, which be divide by the *distance* ***d*** at initial element v_i. In partial entry Nij_{d+1} do must not repeat adjacent pairs, which are presented in preceding partial entry Nij_d, i.e. in its ***j***-row to exclude elements, which be fixed in ***i***-row of preceding partial entry:

i=	*1*	*1*	*1*	*2*	*2*	*2*	*2*	*3*	*3*	*3*	*4*	*5*	*6*	*6*	*7*	*8*	*8*	*9*	*9*	*11*
j=	*2*	*3*	*4*	*3*	*5*	*6*	*7*	*2*	*6*	*7*	*8*	*6*	*5*	*9*	*9*	*11*	*12*	*10*	*11*	*9*
	$Nij_{d=1}$			$Nij_{d=2}$								$Nij_{d=3}$						$Nij_{d=4}$		
				Exclude ***1***								Exclude ***2, 3, 4***						Exclude ***5, 6, 7, 8***		

Comments: **a)** Partial entry $Nij_{d=5}$ do no arise, because ***9*** and ***11*** in it be excluded. **b)** In partial entries Nij_d can be exist also repeated adjacent pairs, for example such as,

2	*3*		*5*	*6*		*9*	*11*	
3	*2*	($Nij_{d=2}$)	*6*	*5*	($Nij_{d=3}$)	*11*	*9*	($Nij_{d=4}$)