

1. SEMIOTIC ATTRIBUTES AND THEIR USING

1.1. Semiotic invariants

We issue from guesswork, that structure of a graph is recognizable by identification its “elementary particles” i.e. by vertex pairs. A vertex pair v_i, v_j is identifiable by its “relation” in the form of a **pair graph** g_{ij} as a sub-graph that represents the *intersection of their neighborhoods*.

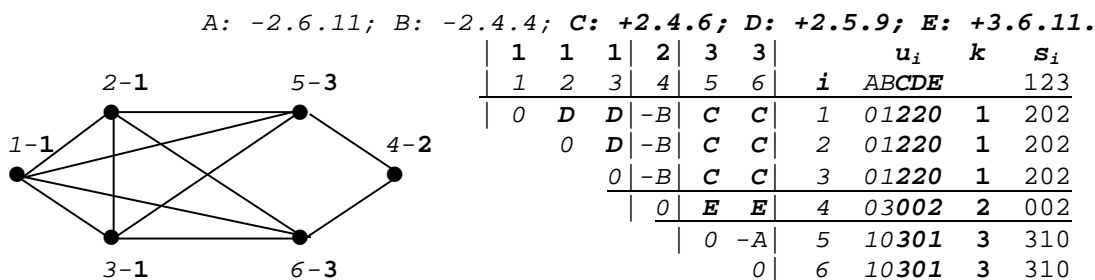
Invariant $\pm d.n.q.ij$ of pair graph is a **semiotic invariant** of structure, where $-d$ signify an usual distance between disadjacent vertices and $+d$ a collateral distance between adjacent vertices v_i and v_j ; n – number of vertices and q – number of edges in the pair graph. Such semiotic invariant we call **pair sign**. A system of pair signs represented in the form of **sign matrix** S . It is treatable as a *text* of graph structure.

The sign matrix S be formed by corresponding Semiotic Identification Principle.

Semiotic Identification Principle. OPERAND: *List the adjacent vertices L.* ALGORITHM: 1) Fix an element i and form its neighborhood N_i , where the elements, connected with i , are divided according to distance d to entries C_d . 2) Fix an element j and form its neighborhood N_j by condition (1). 3) Fix the intersection $N_i \cap N_j$ (as a partial graph) and in therein fix the distance d between i and j , corresponding element set $\{v\}_{ij}$, its power n , the number q of adjacencies (edges). 4) Obtained partial graph is a pair graph g_{ij} and fix its pair sign $\pm d.n.q.ij$. 5) Realize (1) to (4) for each pair ij , $i, j \in [1, |V|]$. 6) Obtained sign matrix S . Fix for each vertex (row) i its *frequency vector* u_i of pair signs. 7) Decompose the sign matrix S by *frequency vectors* u_i lexicographically to partial matrices S_k . 8) In the framework of S_k decompose the rows and columns by class vectors s_i lexicographically to complementary partial matrices S_k . 9) Repeat (8) up to complementary decomposing no arise. RESULTS: a) *Decomposed sign matrix* S ; b) *The lists of vertices* $\{B_{ij}\}$ of pair graphs; c) *Frequency-* u_i and class vectors s_i .

Structure is investigable just by a sign matrix S .

Example 1.1. Results the Semiotic IP. Graph G , its pair signs, decomposed sign matrix S with frequency- and class vectors:



Comment: Sign matrix has recognized three vertex- and five vertex-pair classes, including three edge- and two “non-edge” classes and also the structure as a whole.

Now is the Semiotic IP realized to computer three times: **a)** Performer of first variant was Valdo Praust. This variant works up to the present. **b)** The last variant is very correctly realized by Ashay Dharwadker et al [8].

The pair signs identify the paths, girths, cliques etc attributes of a graph. How to read the pair signs? What are their *structural meanings*?

Propositions 1.1. The structural meanings of *pair signs* $\pm d.n.q.ij$:

P1.1.1. Pair sign in the form $-d.n.q.ij = -\infty.2.0$ identify a *disconnected vertex pair*.

P1.1.2. Pair sign in the form $-d.n.q.ij$ identify a *simple path* or their *assemblage* with length d and we call it *path sign*.

Comments: **a)** Signs of simple path are, for example: $-2.3.2$, identify a 2-path; $-3.4.3$ identify a 3-path; $-7.8.7$ identify a 7-path, etc. **b)** If the length $-d$ no correspond with the numbers of vertices n and edges q in simple path, then is touch with mutually intercrossed d -paths.

P1.1.3. Pair sign with greatest absolute value $\max|-d|$ identify *diameter* of the structure.

P1.1.4. Pair sign in the form $+dnq = +1.2.1$ identify a *link of branch*.

P1.1.5. Pair sign in the form $(+d=2).n.q$ identify *triangularity*, i.e. existence of 3-girths or -cliques.

P1.1.6. Pair sign in the form $(+d=2).n.(q=n(n-1):2)$ identify a *clique* and we call it *clique sign*.

Comments: **a)** Clique signs are, for example: $+1.2.1$, identify a 2-clique; $+2.3.3$ – 3-clique; $+2.4.6$ – 4-clique; $+2.5.10$ – 5-clique; $+2.6.15$ – 6-clique; ..., $+2.13.78$ – 13-clique etc. **b)** Clique sign is a *complete invariant* of the clique, i.e. to clique sign correspond only a clique, to construct a “non-clique” is impossible.

P1.1.7. Pair sign $+dnq$, where $+d > 2$, identify a *girth (cycle)* or their *assemblage* with length $d+1$ and it is called *girth sign*.

Comments: **a)** Girth signs are, for example, $+3.4.4$ for 4-girth; $+4.5.5$ for 5-girth; ... $+7.8.8$ for 8-girth etc. **b)** If the length $+d$ no correspond with the numbers of vertices n and edges q , then is touch with any mutually intercrossed $(d+1)$ -girths.

P1.1.8. Pair sign in the form $(+d=3).(n=a+b).(q=a \times b)$ identify a *bi-clique* with the number of elements a and b in the parts.

Comment: bi-clique sign is, for example, $+3.7.12$, with parts $a=3$ and $b=4$.

P1.1.9. Pair sign $\pm d.n.q.ij$, where n is equal to the number of vertices $|V|$ and q to the number of edges $|E|$ in graph G is a *complete invariant* of G .

Structural meanings of pair signs have essential function by structure research.

There exist graphs where for complete identify need using the *methods of deep identification* (see chapter 3.2) Semiotic invariants are also the *vertex signs*:

Definitions 1.1, Definitions of the *vertex signs*:

D1.1.1. Vector $u_i = u_{i1}, \dots, u_{ip}, \dots, u_{ip}$, whereof elements u_{ip} are lexicographically ordered by pair signs, $d.n.q_1 < \dots < d.n.q_p < \dots < d.n.q_p$, and present their number in a row i of sign matrix S_i called *frequency vector*.

D1.1.2. Vector $s_i = s_{i1}, \dots, s_{ik}, \dots, s_{iK}$, whereof elements s_{ik} present the *number of adjacent vertices in class S_k* of a vertex v_i ; such vector called *class vector*.

1.2. Itemization the regularities

On the ground of sign matrix S can to recognize some well-known and unknown classes of regularity that are here consider and mutually compare. Especially produce the relationships between symmetry properties, strong regularity and bi-, tri- and higher degree cliques.

Definitions 1.2. Definitions of the *regularities* in the graphs:

D1.2.1. Graph, whereof vertices are with equal degree is *(degree)-regular*.

Comment: Graph, where the numbers of pair(+)signs $+dnq$ in all the rows i of sign matrix S are equal is *(degree)-regular*.

D1.2.2. Graph, whereof all the distances between disadjacent vertex pairs are equal is *distance-regular*.

Comments: **a)** Graph, where the partial signs $-d$ of all the pair(-)signs $-dnq$ in sign matrix S are constant is *distance-regular*. **b)** For example, Petersen graph with its pair(-)sign $-2.3.3$ is *2-distance-regular*.

D1.2.3. Graph, whereof all the n vertices belong the same number times to girths with length $n-a$ is *girth-* or *circle-regular*.

Comments: **a)** Graph, where the *girth signs* $+(d>2)nq$ in sign matrix S are equal is *girth-regular*. **b)** For example, Petersen graph with its pair(+)sign $+4.10.15$ is *5-girth-regular* (see Example 1.2).

D1.2.4. Graph, whereof all the n vertices belong the same number times to clique with power $n-b$ is *clique-regular*.

Comments: **a)** Graph, where the numbers of clique signs $+(d=2).n.(q=n(n-1):2)$ in all the rows i of sign matrix S are equal is *clique-regular*. **b)** If *vertex symmetric* graph self not clique, then there cannot exist a single clique, it can be only clique regular. **c)** Unfortunately are almost all the clique algorithms oriented to recognition a *single clique*. For example, in complement of Petersen graph to fix usually a single *4-clique*. In fact is the complement *4-clique-regular*.

D1.2.5. Graph said *strongly regular* with parameters (k,a,b) if it is a *k-degree-regular* incomplete connected graph such that any two adjacent vertices have exactly $a \geq 0$ common neighbors and any two non-adjacent vertices have $b \geq 1$ common neighbors.

Comments: **a)** Common neighbors can be have only *2-distance regular* graphs with constant n , i.e. $-2.(n=const).q$, where $n-2$ is the number of common neighbors. **b)** For example, pair signs of a Weisfeiler's strongly regular graph are $-2.8.20$, $-2.8.19$, $-2.8.18$ and $+2.7.13$, $+2.7.14$, $+2.7.15$ (Example 3.1). **c)** Petersen graph is bisymmetric and *strongly regular*.

1.3. Girth- and clique regularity

Clique recognition is a popular task of graph theory. Girth recognition for some reason or other not.

Ideology of almost all clique recognition algorithms be grounded on the distinction the vertices that belong to a greatest clique, from others. Methods for that purpose are elaborated in gross.

Example 1.2. Sign matrices with the lists of the girths and partial cliques of Petersen graph *PET* and its complement *PETC*:

$$A: -2.3.2; B: +4.10.15.$$

$$A: -2.6.12; B: +2.5.8.$$

1	2	3	4	5	6	7	8	9	10	<i>i</i>	<i>AB</i>	<i>Orb</i>	1	2	3	4	5	6	7	8	9	10	<i>AB</i>
0	<i>B</i>	- <i>A</i>	- <i>A</i>	<i>B</i>	<i>B</i>	- <i>A</i>	- <i>A</i>	- <i>A</i>	- <i>A</i>	1	63	1	0	- <i>A</i>	<i>B</i>	<i>B</i>	- <i>A</i>	- <i>A</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>	36
	0	<i>B</i>	- <i>A</i>	- <i>A</i>	- <i>A</i>	<i>B</i>	- <i>A</i>	- <i>A</i>	- <i>A</i>	2	63	1		0	- <i>A</i>	<i>B</i>	<i>B</i>	<i>B</i>	- <i>A</i>	<i>B</i>	<i>B</i>	<i>B</i>	36
		0	<i>B</i>	- <i>A</i>	- <i>A</i>	- <i>A</i>	<i>B</i>	- <i>A</i>	- <i>A</i>	3	63	1			0	- <i>A</i>	<i>B</i>	<i>B</i>	<i>B</i>	- <i>A</i>	<i>B</i>	<i>B</i>	36
			0	<i>B</i>	- <i>A</i>	- <i>A</i>	- <i>A</i>	<i>B</i>	- <i>A</i>	4	63	1				0	- <i>A</i>	<i>B</i>	<i>B</i>	<i>B</i>	- <i>A</i>	<i>B</i>	36
				0	- <i>A</i>	- <i>A</i>	- <i>A</i>	- <i>A</i>	<i>B</i>	5	63	1					0	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>	- <i>A</i>	36
					0	- <i>A</i>	<i>B</i>	<i>B</i>	- <i>A</i>	6	63	1						0	<i>B</i>	- <i>A</i>	- <i>A</i>	<i>B</i>	36
						0	- <i>A</i>	<i>B</i>	<i>B</i>	7	63	1							0	<i>B</i>	- <i>A</i>	- <i>A</i>	36
							0	- <i>A</i>	<i>B</i>	8	63	1								0	<i>B</i>	- <i>A</i>	36
								0	- <i>A</i>	9	63	1									0	<i>B</i>	36
									0	10	63	1										0	36

Comments:

- Petersen graph is *bisymmetric* and *strongly regular*. with $a=0$ and $b=1$.
- From sign matrix is gather from reading that Petersen graph *PET* has exactly twelve 5-girths, in present case: 1-2-3-4-5-1 and 6-8-10-7-9-6 and 1-2-3-8-6-1 and 1-2-7-10-5-1 and 1-5-4-9-6-1 and 2-3-4-9-7-2 and 3-4-5-10-8-3 and 1-2-7-9-6-1 and 1-5-10-8-6-1 and 2-3-8-10-7-2 and 3-4-9-6-8-3 and 4-5-10-7-9-4. Each vertex belongs to six 5-girths. Each edge belongs to four 5-girths. It is *5-girth regular*.
- The complement of Petersen graph *PETC* has five 4-cliques, in present case: 1,3,9,10 and 2,4,6,10 and 1,4,7,8 and 2,5,8,9 and 3,5,6,7. Each vertex belongs to two cliques. Each edge belongs to one clique. It is *4-clique regular*.
- Pair sign +4.10.15 signify that graph consists of 10 vertices and 15 edges that form 5-girths – it is a *complete invariant* of this graph. Pair sign -2.3.2 fix the paths with length 2 that exists between disadjacent vertices of 5-girths. If set up to construct a degree regular graph with 10 vertices, such that its 15 edges form 5-girths, we become only the Petersen graph.

If clique has a meaning and if it is findable then usually is it educe. In case of clique regularity so no done, even though touch with a warrantable phenomenon.

Propositions 1.2. Relationships between *symmetry, girth- and clique regularity*:

P1.2.1. All the *vertex symmetric* (“transitive”) graphs are even if *girth regular* or *clique regular*, or *girth- and clique regular* – and contrariwise.

Comments: All the girth regular and/or clique regular graphs are vertex symmetric.

P1.2.2. The complement of an *edge-symmetric girth regular* graph is *clique regular*.

Comments: a) It is showed on Examples 1.2, 1.3 and 1.5. b) Is *triangle* a clique or a girth?

P1.2.3. A *vertex-symmetric* graph can be at the same time so *girth- and clique regular*.

Comment: It is showed on Example 3.3.

Example 1.3. Sign matrices of dodecahedron *DOD* and its complement *DODC*:

$$-A=-5.20.30; -B=-4.8.9; -C=-3.4.3; -D=-2.3.2; +E=+4.8.9.$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	<i>i</i>	<i>ABCDE</i>	<i>Orb</i>
0	<i>E</i>	- <i>D</i>	- <i>C</i>	- <i>B</i>	- <i>C</i>	- <i>D</i>	<i>E</i>	- <i>D</i>	- <i>C</i>	- <i>B</i>	- <i>A</i>	- <i>B</i>	- <i>C</i>	- <i>D</i>	- <i>D</i>	- <i>C</i>	- <i>C</i>	- <i>D</i>	<i>E</i>	1	13663	1
	0	<i>E</i>	- <i>D</i>	- <i>C</i>	- <i>C</i>	- <i>D</i>	- <i>D</i>	- <i>C</i>	- <i>B</i>	- <i>A</i>	- <i>B</i>	- <i>C</i>	- <i>D</i>	- <i>E</i>	- <i>D</i>	- <i>C</i>	- <i>B</i>	- <i>C</i>	- <i>D</i>	2	13663	1
		0	<i>E</i>	- <i>D</i>	- <i>D</i>	<i>E</i>	- <i>D</i>	- <i>C</i>	- <i>C</i>	- <i>B</i>	- <i>C</i>	- <i>C</i>	- <i>D</i>	- <i>D</i>	- <i>C</i>	- <i>B</i>	- <i>A</i>	- <i>B</i>	- <i>C</i>	3	13663	1
			0	<i>E</i>	- <i>D</i>	- <i>D</i>	- <i>C</i>	- <i>B</i>	- <i>C</i>	- <i>C</i>	- <i>D</i>	- <i>D</i>	<i>E</i>	- <i>D</i>	- <i>C</i>	- <i>C</i>	- <i>B</i>	- <i>A</i>	- <i>B</i>	4	13663	1
				0	<i>E</i>	- <i>D</i>	- <i>C</i>	- <i>C</i>	- <i>D</i>	- <i>D</i>	<i>E</i>	- <i>D</i>	- <i>D</i>	- <i>C</i>	- <i>B</i>	- <i>C</i>	- <i>C</i>	- <i>B</i>	- <i>A</i>	5	13663	1

0	E	-D	-D	E	-D	-D	-C	-C	-B	-A	-B	-C	-C	-B		6	13663	1
0	E	-D	-D	-C	-C	-B	-C	-C	-B	-A	-B	-C	-C		7	13663	1	
0	E	-D	-C	-B	-A	-B	-C	-C	-B	-C	-D	-D		8	13663	1		
0	E	-D	-C	-B	-A	-B	-C	-C	-D	E	-D		9	13663	1			
0	E	-D	-C	-B	-A	-B	-C	-D	-D		10	13663	1					
0	E	-D	-C	-B	-C	-D	E	-D	-C		11	13663	1					
0	E	-D	-C	-C	-D	-D	-C	-B		12	13663	1						
0	E	-D	-D	E	-D	-C	-C		13	13663	1							
0	E	-D	-D	-C	-B	-C		14	13663	1								
0	E	-D	-C	-C	-D		15	13663	1									
0	E	-D	-D	E		16	13663	1										
0	E	-D	-D		17	13663	1											
0	E	-D		18	13663	1												
0	E		19	13663	1													
0		20	13663	1														

-A=-2.16.102; +B=+2.14.78; +C=+2.14.79; +D=+2.15.89.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	i	ABCCD	Orb
0	-A	D	B	C1	B	D	-A	D	B	C1	C2	C1	B	D	D	B	B	D	-A	1	36316	1
0	-A	D	B	B	D	D	B	C1	C2	C1	B	D	-A	D	B	C1	B	D		2	36316	1
0	-A	D	D	-A	D	B	B	C1	B	B	D	D	B	C1	C2	C1	B		3	36316	1	
0	-A	D	D	B	C1	B	B	D	D	-A	D	B	B	C1	C2	C1	B		4	36316	1	
0	-A	D	B	B	D	D	-A	D	D	B	C1	B	B	C1	C2	B		5	36316	1		
0	-A	D	D	-A	D	D	B	B	C1	C2	C1	B	B	C1	B	B	C1		6	36316	1	
0	-A	D	D	B	C1	B	B	C1	C2	C1	B	B	C1	C2	C1	B	B		7	36316	1	
0	-A	D	B	C1	C2	C1	B	B	C1	B	D	D		8	36316	1						
0	-A	D	B	C1	C2	C1	B	B	D	-A	D		9	36316	1							
0	-A	D	B	C1	C2	C1	B	D	D	B		10	36316	1								
0	-A	D	B	C1	B	D	-A	D	B		11	36316	1									
0	-A	D	B	D	D	B	C1		12	36316	1											
0	-A	D	D	-A	D	B	B	D	B	B	D		13	36316	1							
0	-A	D	D	B	C1	B	B	C1	C2	C1	B	B	D		14	36316	1					
0	-A	D	D	B	B	D	-A	D		15	36316	1										
0	-A	D	D	-A	D	B	B	D	-A	D		16	36316	1								
0	-A	D	D	-A	D	D	-A	D	B	B	D		17	36316	1							
0	-A	D	D	-A	D	D	-A	D	D		18	36316	1									
0	-A	D	D	-A	D	D	-A	D	B	B	D		19	36316	1							
0		20	36316	1																		

Comments:

- a) Graph *DOD* is (+)- or *edge symmetric* and its complement *DODC* is (-)- or “*non-edge symmetric*”.
- b) *DOD* is *5-girth-* and *3-degree regular* and its complement *DODC* is *2-distance-* and *16-degree regular*.

Is the complement of *5-girth regular edge symmetric* dodecahedra according to P1.2.2 *clique regular*? Semiotic recognition the cliques take place by the *clique signs*. If such in sign matrix *S* no exist, then emerge these in *local sign matrices S_{ij}* of corresponding pair(+)graphs *g_{ij}*.

Example 1.4. In complement *DODC* the explicit clique signs no exist. But in the local sign matrices, that obtained by processing the pair graphs with signs *+B=+2.14.78* contain *8-clique signs +2.8.28*. On the ground of such local sign matrices *W_{1,4}*, *W_{5,9}*, *W_{3,16}*, *W_{6,13}* ja *W_{5,8}* can be to recognize all the partial cliques of *8-clique regular graph DODC*:

<i>i=</i>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I	•			•			•			•		•			•		•		•	
II		•			•		•		•		•		•			•		•		•
III	•		•			•			•			•		•		•		•		•
IV		•		•		•		•		•			•			•		•		•
V			•		•			•		•			•		•			•		•

Comments: a) The *partial 8-cliques* of 8-clique regular **DODC** cover all the 20 vertices. b) All five partial cliques are *intercrossed*, where all the 10 edges belong to pair orbit **C2**. c) The edges of **C2** make each partial clique to intercross with residual four partial cliques:

$i-j=$	1-12	2-11	3-18	4-19	5-20	6-16	7-17	8-13	9-14	10-15
Osaklikk	I	II	III	I	II	III	I	IV	II	I
Osaklikk	III	IV	V	IV	V	IV	II	V	III	V

Form known graphs are *clique regular* also complements of Heawood's, Coxeter's, Folkman's graphs. Their originals are bipartite and by all the nature laws represent the complements of such parts self-evidently cliques.

Proposition 1.3. On the *partial cliques*:

P1.3.1. Partial cliques of a clique regular graph can be *disconnected partial, mutually connected or intercrossed*.

Comments: Most of the clique recognition algorithms are in the case of clique regularity unworkable, the others produce *only one clique*.

P1.3.2. Complement of a *m-partite* graph in case of equal *n* parts is *n-clique regular*, with the number *m* of non-intercrossed *n-cliques*.

Example 1.5. Sign matrices of Heawood's graph **HEA** and its complement **HEAC**:

$$A: -3.8.9; B: -2.3.2; C: +5.14.21.$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	i	ABC	Orb	deg
0	C	-B	-A	-B	C	-B	-A	-B	-A	-B	-A	-B	C	1	463	1	3
	0	C	-B	-A	-B	-A	-B	-A	-B	C	-B	-A	-B	2	463	1	3
		0	C	-B	-A	-B	C	-B	-A	-B	-A	-B	-A	3	463	1	3
			0	C	-B	-A	-B	-A	-B	-A	-B	C	-B	4	463	1	3
				0	C	-B	-A	-B	C	-B	-A	-B	-A	5	463	1	3
					0	C	-B	-A	-B	-A	-B	-A	-B	6	463	1	3
						0	C	-B	-A	-B	C	-B	-A	7	463	1	3
							0	C	-B	-A	-B	-A	-B	8	463	1	3
								0	C	-B	-A	-B	C	9	463	1	3
									0	C	-B	-A	-B	10	463	1	3
										0	C	-B	-A	11	463	1	3
											0	C	-B	12	463	1	3
												0	C	13	463	1	3
													0	14	463	1	3

$$A: -2.10.36; B: +2.8.22; C: +2.9.30.$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	i	ABC	Orb	deg
0	-A	C	B	C	-A	C	B	C	B	C	B	C	-A	1	346	1	10
	0	-A	C	B	C	B	C	B	C	-A	C	B	C	2	346	1	10
		0	-A	C	B	C	-A	C	B	C	B	C	B	3	346	1	10
			0	-A	C	B	C	B	C	B	C	-A	C	4	346	1	10
				0	-A	C	B	C	-A	C	B	C	B	5	346	1	10
					0	-A	C	B	C	B	C	B	C	6	346	1	10
						0	-A	C	B	C	-A	C	B	7	346	1	10
							0	-A	C	B	C	B	C	8	346	1	10
								0	-A	C	B	C	-A	9	346	1	10
									0	-A	C	B	C	10	346	1	10
										0	-A	C	B	11	346	1	10
											0	-A	C	12	346	1	10
												0	-A	13	346	1	10
													0	14	346	1	10

Comments:

- a) Graph **HEA** is *edge symmetric* and *bipartite*, where its parts in present case divide to vertices with even numbers and vertices with odd numbers. It is also **6-girth-** and **3-degree regular**.
- b) **HEA** is *structurally unique*, the pair sign **+5.14.21** signify that its 14 vertices form 21 adjacent pairs that belong to 6-girths and form a *complete invariant* of this graph.
- c) From bipartite **HEA** conclude that its complement **HEAC** consist of two *mutually connected 7-cliques*, it is **7-clique regular**, where the cliques correspond to the parts of **HEA**. **HEAC** is also (-)*symmetric*, **2-distance-** and **10-degree regular**.

Example 1.6. Sign matrix the graph **KOHC** (complement of **KOH** on Example 2.2):

$$-A=-2.8.15; -B=-2.7.13; +C=+2.4.5; +D=+2.5.10; +E=+2.7.14.$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	i	ABCDE	orb	deg
0	-B	C	D	-B	-A	E	C	C	E	-A	-B	D	C	-B	1	24422	1	8
	0	-B	C	D	-B	-A	E	C	C	E	-A	-B	D	C	2	24422	1	8
		0	-B	C	D	-B	-A	E	C	C	E	-A	-B	D	3	24422	1	8
			0	-B	C	D	-B	-A	E	C	C	E	-A	-B	4	24422	1	8
				0	-B	C	D	-B	-A	E	C	C	E	-A	5	24422	1	8
					0	-B	C	D	-B	-A	E	C	C	E	6	24422	1	8
						0	-B	C	D	-B	-A	E	C	C	7	24422	1	8
							0	-B	C	D	-B	-A	E	C	8	24422	1	8
								0	-B	C	D	-B	-A	E	9	24422	1	8
									0	-B	C	D	-B	-A	10	24422	1	8
										0	-B	C	D	-B	11	24422	1	8
											0	-B	C	D	12	24422	1	8
												0	-B	C	13	24422	1	8
													0	-B	14	24422	1	8
														0	15	24422	1	8

Comments: a) Graph **KOHC** is *poly symmetric*. b) It has clique sign **D=+2.5.10** and is **5-clique-** and **2-distance regular** and *triangular*:

Nr.	Clique signs D	Partial 5-cliques
1	1-4, 1-13, 4-7, 7-10, 10-13	1,4,7,10,13
2	2-5, 2-14, 5-8, 8-11, 11-14	2,5,8,11,14
3	3-6, 3-15, 6-9, 9-12, 12-15	3,6,9,12,15

c) Edges that correspond to signs **+C** and **+E** make the partial 5-cliques to *mutually connected*.

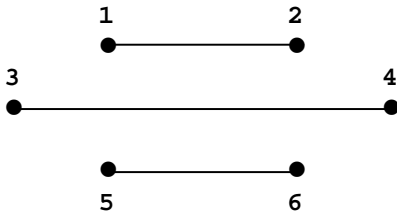
1.4. Bisymmetry, clique- and strong regularity

A graph is *bisymmetric* if it has only two pair signs. There exist an interest relationship between bisymmetry, clique- and strong regularity. Bisymmetric graph has just one edge orbit and one “non-edge” orbit.

NB! Not forget that the *complement* of bisymmetric graph is also bisymmetric.

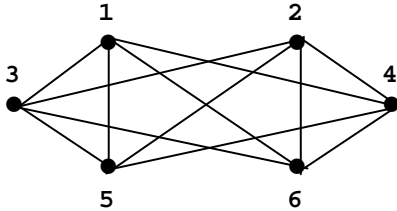
Example 1.7. Graph **B6-3**, its complement **B6-12**, their pair signs, sign matrices and common symmetry characteristics:

$$A:-0.2.0; B:+1.2.1.$$



	1	2	3	4	5	6	i	AB	deg
1	0	B	$-A$	$-A$	$-A$	$-A$	1	41	1
2		0	$-A$	$-A$	$-A$	$-A$	2	41	1
3			0	B	$-A$	$-A$	3	41	1
4				0	$-A$	$-A$	4	41	1
5					0	B	5	41	1
6						0	6	41	1

$A: -2.6.12; B: +2.4.5.$



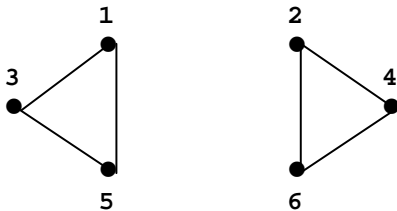
	1	2	3	4	5	6	i	AB	deg
1	0	$-A$	B	B	B	B	1	14	4
2		0	B	B	B	B	2	14	4
3			0	$-A$	B	B	3	14	4
4				0	B	B	4	14	4
5					0	$-A$	5	14	4
6						0	6	14	4

SRV	HR	SR	aut
$3^1 12^1$	0.2173	0.8152	48

Comments a) Graph **B6-3** and its complement **B6-12** are *bisymmetric*. b) Graph **B6-3** consist of *three disconnected partial 2-cliques*, it is *2-clique regular*. c) Complement **B6-12** is *three partite*, where its parts correspond to 2-cliques of **B6-3**. It is a so called *partite clique*, exactly with a *2-tri-clique*, generally called *n-m-clique*. It is simply sight that all the vertices belong to *triangles*.

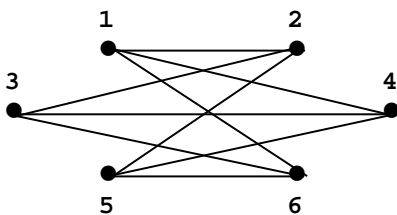
Example 1.8. Graph **B6-6**, its complement **B6-9**, their pair signs, sign matrices and common symmetry characteristics:

$A: -0.2.0; B: +2.3.3.$



	1	2	3	4	5	6	i	ABC	deg
1	0	$-A$	B	$-A$	B	$-A$	1	32	2
2		0	$-A$	B	$-A$	B	2	32	2
3			0	$-A$	B	$-A$	3	32	2
4				0	$-A$	B	4	32	2
5					0	$-A$	5	32	2
6						0	6	32	2

$A: -2.5.6; B: +3.6.9.$



	1	2	3	4	5	6	i	AB	deg
1	0	B	$-A$	B	$-A$	B	1	23	3
2		0	B	$-A$	B	$-A$	2	23	3
3			0	B	$-A$	B	3	23	3
4				0	B	$-A$	4	23	3
5					0	B	5	23	3
6						0	6	23	3

SRV	HR	SR	aut
$6^1 9^1$	0.2923	0.7515	72

Comments a) Graph **B6-6** and its complement **B6-9** are *bisymmetric*. b) Graph **B6-6** consist of *two disconnected partial 3-cliques*, it is *3-clique regular*. c) Complement **B6-9** is *bipartite*, where its parts correspond to 3-cliques of **B6-6**. d) **B6-9** is a *3-bi-clique* that is *4-girth regular*. e) Pair sign $+3.6.9$ cover all the $n=6$ vertices and all the $q=9$ edges, it is the *complete invariant* of **B6-9**.

Generally, the interest for structural properties of graphs is not much. In case of practical tasks these no crop up. But yet is touch with true lawfulness.

Propositions 1.4. The relationships between *bisymmetry and partial cliques*:

P1.4.1. The *complement* of a graph with m equal *disconnected partial cliques* is a *bisymmetric m -partite complete graph*, i.e. it is a *n - m -clique* – and contrariwise.

Size	Graph	Its complement
m	Number of disconnected partial cliques	Number of parts
n	Power of disconnected partial cliques	Power of parts

Comments: **a)** For example, the complement of the structure with two disconnected partial cliques is a *bi-clique*, with three disconnected partial cliques is a *tri-clique*, etc. **b)** On the aspect of bisymmetry no essential is the structure *disconnected partial* or *connected*.

P1.4.2. All the *n - m -cliques* with equal power n and their complements are *bisymmetric*.

P1.4.3. A *n - m -clique* contain an usual clique with the power m , it is *m -clique regular*.

Comment: For example, bi-clique is *2-clique regular*, tri-clique is *3-clique regular*, etc.

P1.4.4. A bisymmetric *n - m -clique* contain $s=n^m$ usual cliques with power m .

P1.4.5. The number of edges $|E|$ of bisymmetric *n - m -clique* equal to the product of quadrate of power n^2 of parts and the number m of edges in the usual clique:

<i>n-m-clique</i>			
Symmetry	Power of cliques	Number of cliques	Number of edges
<i>Bisymmetry</i>	m	n^m	$E=n^2m(m-1):2$

P1.4.6. Correspondingly to the number of parts we call a *n - m -clique* to *bi-, tri-, quadro-, quinta-, sexta-, septa-, octa-, nona-, deca-, or undeca-* etc *-clique*.

Comments: Among the graphs with 1 to 20 vertices there exist exactly following *n - m -cliques*:

One *n - m -clique* with 4-vertces – *2-bi-clique* as a complement of disconnected partial 2-cliques;

Two *n - m -cliques* with 6 vertices – *2-tri-clique* and *3-bi-clique* as complements of disconnected partial 2- and 3-cliques correspondingly;

Two *n - m -cliques* with 8 vertices – *2-quadro-clique* and *4-bi-clique* as complements of disconnected partial 2- and 4-cliques correspondingly;

One *n - m -clique* with 9 vertices – *3-tri-clique* as a complement of disconnected partial 3-cliques correspondingly;

Two *n - m -cliques* with 10 vertices – *2-quinta-* and *5-bi-clique* as complements of disconnected partial 2- and 5-cliques correspondingly;

Four *n - m -cliques* with 12 vertices – *2-sexta-, 3-quadro-, 4-tri-* and *6-bi-clique* as complements of disconnected partial 2-, 3-, 4- and 6-cliques correspondingly;

Two *n - m -cliques* with 14 vertices – *2-septa-* and *7-bi-clique* as complements of disconnected partial 2- and 7-cliques correspondingly;

Two *n - m -cliques* with 15 vertices – *3-quinta-* and *5-tri-clique* as complements of disconnected partial 3- and 5-cliques correspondingly;

Three *n - m -cliques* with 16 vertices – *2-octa-, 4-quadro-* and *8-bi-clique* as complements of disconnected partial 2-, 4- and 8-cliques correspondingly;

Four *n - m -cliques* with 18 vertices – *2-nona-, 3-sexta-, 6-tri-* and *9-bi-clique* as complements of disconnected partial 2-, 3-, 6- and 9-cliques correspondingly;

Four n - m -cliques with 20 vertices – *2-deca-*, *4-quinta-*, *5-quadro-* and *10-bi-clique* as complements of disconnected partial 2-, 4-, 5- and 10-cliques correspondingly;

In all **27 n - m -cliques**.

Thus, the n - m -cliques are simply constructable.

By D1.2.5 a graph said **strongly regular** with parameters (k,a,b) if it is a k -regular incomplete and connected graph such that any two adjacent vertices have exactly $a \geq 0$ common neighbors and any two non-adjacent vertices have $b \geq 1$ common neighbors.

Propositions 1.5. The properties that are connected with **strong regularity**:

P1.5.1. All the **connected bisymmetric** 2-distance regular structures are **strongly regular**, but no on the contrary.

Comments: a) Existence in connected bisymmetric structure exactly two different pair signs, – $d.n_1.q$ and $+d.n_2.q$, mean that by $\pm d=2$ has each disadjacent vertex pair exactly n_1-2 common neighbors and each adjacent vertex pair n_2-2 common neighbors. In case $+d>2$ no exist common neighbors. b) The numbers $n-2$ of common neighbors can be stay constant also by existence more that two pair signs, i.e. by mono-, multi- and local symmetries. Consequently, strongly regular graphs can be also *mono-, multi- and partial symmetric*.

P1.5.2. All the n - m -cliques are **strongly regular**, but no on the contrary.

P1.5.3. The **connected complement** of a strongly regular structure is also **strongly regular**.

In addition of simply constructable n - m -cliques are recognized following bisymmetric-strongly regular structures:

1) self-complemented 5-girth; 2) self-complemented B9-18; 3) Petersen graph B10-15; 4) and its complement B10-30; 5) self-complemented B13-39; 6) Weisfeiler's B15-45; 7) and its complement B15-60; 8) Greenwood's (Clebish's) B16-40; 9) and its complement B16-80; 10) Shrikhande B16-48; 11) and its complement B16-72; 12) self-complemented B17-68.

Consequently, among the structures with 1 to 20 vertices there exists $27+12=39$ bisymmetric + strongly regular + clique- or girth regular graphs

Example 3.8. List of all the 39 *bisymmetric + strongly regular + clique- or girth regular* graphs with 1 to 20 vertices:

Nr	Notation	deg	SRV	SR	Cmp/prt		Regularity	Number	Commentary	Pair signs	
					m	n				Pair(-) sign	Pair(+) sign
1	B4-4	2	$2^1 4^1$	0.6448	2p	2	4-girth	-	2-bi-clique	-2.4.4	+3.4.4
2	B5-5	2	5^2	0.6990	1c	5	5-girth	-	Selfcomplem.	-2.3.2	+4.5.5
3	B6-12	4	$3^1 12^1$	0.8152	3p	2	3-clique	8	2-tri-clique	-2.6.12	+2.4.5
4	B6-9	3	$6^1 9^1$	0.7515	2p	3	4-girth	-	3-bi-clique	-2.5.6	+3.6.9
5	B8-24	6	$4^1 24^1$	0.8769	4p	2	4-clique	16	2-quadro-clique	-2.8.24	+2.6.13
6	B8-16	4	$12^1 16^1$	0.7906	2p	4	4-girth	-	4-bi-clique	-2.6.8	+3.8.16
7	B9-27	6	$9^1 27^1$	0.8431	3p	3	3-clique	27	3-tri-clique	-2.8.21	+2.5.7
8	B9-18	4	18^2	0.8066	3p	3	3-girth	6	Selfcomplem.	-2.4.4	+2.3.3
9	B10-40	8	$5^1 40^1$	0.9084	5p	2	5-clique	32	2-quinta-clique	-2.10.40	+2.8.25
10	B10-15	3	$15^1 30^1$	0.8328	1c	10	5-girth	12	Petersen gr.	-2.3.2	+4.10.15
11	B10-30	6			1c	10	4-clique	5	Petersen comp.	-2.6.12	+2.5.8
12	B10-25	5	$20^1 25^1$	0.8196	2p	5	4-girth	-	5-bi-clique	-2.7.10	+3.10.25

13	B12-60	10	<u>6¹60¹</u>	<u>0.9273</u>	6p	2	6-clique	64	<u>2-sexta-clique</u>	<u>-2.12.60</u>	<u>+2.10.41</u>
14	B12-54	9	<u>12¹54¹</u>	<u>0.8868</u>	4p	3	4-clique	81	<u>3-quadro-clique</u>	<u>-2.11.45</u>	<u>+2.8.22</u>
15	B12-48	8	<u>18¹48¹</u>	<u>0.8601</u>	3p	4	3-clique	64	<u>4-tri-clique</u>	<u>-2.10.32</u>	<u>+2.6.9</u>
16	B12-36	6	<u>30¹36¹</u>	<u>0.8355</u>	2p	6	4-girth	-	<u>6-bi-clique</u>	<u>-2.8.12</u>	<u>+3.12.36</u>
17	B13-39	6	<u>39²</u>	<u>0.8409</u>	1c	1	3-clique	22	<u>Selfcomplem.</u>	<u>-2.5.7</u>	<u>+2.4.5</u>
18	B14-84	12	<u>7¹84¹</u>	<u>0.9399</u>	7p	2	7-clique	128	<u>2-septa-clique</u>	<u>-2.14.84</u>	<u>+2.12.61</u>
19	B14-49	7	<u>42¹49¹</u>	<u>0.8470</u>	2p	7	4-girth	-	<u>7-bi-clique</u>	<u>-2.9.14</u>	<u>+3.14.49</u>
20	B15-90	12	<u>15¹90¹</u>	<u>0.9119</u>	5p	3	5-clique	243	<u>3-quinta-clique</u>	<u>-2.14.78</u>	<u>+2.11.46</u>
21	B15-75	10	<u>30¹75¹</u>	<u>0.8711</u>	3p	5	3-clique	125	<u>5-tri-clique</u>	<u>-2.12.45</u>	<u>+2.7.11</u>
22	B15-45	6	<u>45¹60¹</u>	<u>0.8533</u>	1c	15	3-clique	-	<u>Weisfeiler</u>	<u>-2.5.6</u>	<u>+2.3.3</u>
23	B15-60	8			1c	15	5-clique	-	<u>Weisfeil. comp.</u>	<u>-2.6.12</u>	<u>+2.6.12</u>
24	B16-112	14	<u>8¹112¹</u>	<u>0.9488</u>	8p	2	8-clique	256	<u>2-octa-clique</u>	<u>-2.16.112</u>	<u>+2.14.85</u>
25	B16-96	12	<u>24¹96¹</u>	<u>0.8955</u>	4p	4	4-clique	256	<u>4-quadro-clique</u>	<u>-2.14.72</u>	<u>+2.10.33</u>
26	B16-40	5	<u>40¹80¹</u>	<u>0.8670</u>	4p	4	4-girth	-	<u>Greenwood</u>	<u>-2.4.4</u>	<u>+3.10.13</u>
27	B16-80	10			1c	16	5-clique	16	<u>Greenw. comp.</u>	<u>-2.8.24</u>	<u>+2.8.22</u>
28	B16-48	6	<u>48¹72¹</u>	<u>0.8594</u>	1c	16	4-clique	-	<u>Shrikhande</u>	<u>-2.4.4</u>	<u>+2.4.6</u>
29	B16-72	9			1c	16	4-clique	-	<u>Shrikhan comp.</u>	<u>-2.8.18</u>	<u>+2.6.11</u>
30	B16-64	8	<u>56¹64¹</u>	<u>0.8557</u>	2p	8	4-girth	-	<u>8-bi-clique</u>	<u>-2.10.10</u>	<u>+3.16.64</u>
31	B17-68	8	<u>68²</u>	<u>0.8589</u>	1c	17	3-clique	-	<u>Selfcomplem.</u>	<u>-2.6.11</u>	<u>+2.5.7</u>
32	B18-144	16	<u>9¹144¹</u>	<u>0.9555</u>	9p	2	9-clique	512	<u>2-nona-clique</u>	<u>-2.18.144</u>	<u>+2.16.113</u>
33	B18-135	15	<u>18¹135¹</u>	<u>0.9280</u>	6p	3	6-clique	729	<u>3-sexta-clique</u>	<u>-2.17.120</u>	<u>+2.14.79</u>
34	B18-108	12	<u>45¹108¹</u>	<u>0.8796</u>	3p	6	3-clique	216	<u>6-tri-clique</u>	<u>-2.14.60</u>	<u>+2.8.13</u>
35	B18-81	9	<u>72¹81¹</u>	<u>0.8626</u>	2p	9	4-girth	-	<u>9-bi-clique</u>	<u>-2.11.18</u>	<u>+3.18.81</u>
36	B20-180	18	<u>10¹180¹</u>	<u>0.9607</u>	10p	2	10-clique	1036	<u>2-deca-clique</u>	<u>-2.20.180</u>	<u>+2.18.45</u>
37	B20-160	16	<u>30¹160¹</u>	<u>0.9169</u>	5p	4	5-clique	1924	<u>4-quinta-clique</u>	<u>-2.18.128</u>	<u>+2.14.73</u>
38	B20-150	15	<u>40¹150¹</u>	<u>0.9019</u>	4p		4-clique	625	<u>5-quadro-clique</u>	<u>-2.17.105</u>	<u>+2.12.46</u>
39	B20-100	10	<u>90¹100¹</u>	<u>0.8682</u>	2p	10	4-girth	-	<u>10-bi-clique</u>	<u>-2.12.20</u>	<u>+3.20.100</u>

Comments: **a)** The marking of structure show the numbers of vertices and edges. **b) deg** – degree. **c) SRV** – symmetry vector (Prop. 2.5); **d) SR** – symmetry value (Prop. 2.7). **e) c** – number of components. **f) m** – number of parts. **g) n** – power of parts. **h) s** – number of cliques.

Proposition 1.6. Pair(+)sign of a *bi-clique* and pair(-)sign of a *2-m-clique* are *complete invariants* of the structure (in the table are these underlined).

Usually are the *lists of strongly regular graphs* incomplete. For example, in the list [9] lack 31 bisymmetric-strongly regular structures with 4 to 20 vertices.

In the “most complete” list [10] where are give 33 structures, among these also *n-m-cliques* lack unfortunately **25** (B16-96), **29** (B16-72), **33** (B18-135), **34** (B18-108), **37** (B20-160) and **38** (B20-150).

In such lists exists also many large graphs. For example, is the first list to find a graph with 999 vertices:

Example 1.10. We can in a simple way to induce some bisymmetric, clique- and strongly regular graphs with 999 vertices. In the lists of strongly regular graphs cannot these to find:

Nr	Notation	deg	E	SR	Regularity	Commentary	(+)signs
1	B999-2	2	999	0.9989	3-clique	333 disconnected partial 3-cliques	+2.3.3
2	B999-996	996	497502		333-clique	333 3-elementic parts 3-tricent-triginta-tri-clique	?
3	B999-8	8	3996	0.9979	9-clique	111 disconnected partial 9-cliques	+2.9.36
4	B999-990	990	494505		111-clique	111 9-elementic parts 9-cent-undeca-clique	?

5	B999-110	110	54945		<i>111-clique</i>	9 disconnected partial 111-cliques	+2. 111.6105
6	B999-888	888	443556	0.9736	<i>9-clique</i>	9 111-elementic parts 111-nona-clique	?
7	B999-332	332	165832		<i>333-clique</i>	3 disconnected partial 333-cliques	+2. 333.55278
8	B999-666	666	332667	0.9515	<i>3-clique</i>	3 333-elementic parts 333-tri-clique	?

Comments: a) Strongly regular are there only *n-m*-cliques. b) The names of *n-m*-cliques can be for any no please, but others I cannot find.

Proposition 3.7. Semiotic approach discovers some new strongly- and clique regular structures.

Summary

So it is recognized 39 bisymmetric-strongly regular structures with 4 to 20 vertices, mainly on the ground of disconnected partial cliques induced structures. The results of J. Petersen (B10-15), A. Titov (B13-39), B. Weisfeiler (B15-45) et al, Greenwood-Gleason (or Clebish B16-40) in the realm of bisymmetry are random coincides, because the first be interested on valence-regularity, other on self-complementary, third on strong regularity, fourth on color-conjecture, others on isomorphism testing etc.

In the “most complete” list of strongly regular graphs are showed all the to 20 vertices *bi-cliques*, as *complete bipartite graphs*, whereby bi-clique with 4 vertices called *square* and with 6 vertices called *unity*. There are also showed all the *2-m-cliques*, that have title *r-cocktail party graphs*, whereby with 6 vertices called *octahedral graph* and with 8 vertices *16-cell graph*. Other *n-m-cliques* called mostly *circular graphs*. There lack five *n-m-cliques* and the complement of a known strongly regular graph

It is touch with partial coincide the bisymmetry and strong regularity. Bisymmetry cover also disconnected structures and strong regularity can be exists also by mono-, poly- and partial symmetry. But the lasts no exist among the structures with to 20 vertices. Semiotic approach was fill the “white blotch” of lists the strongly regular graphs, was pick out the essence of so far ignored clique regularity, and this that the complement of strongly regular graph is also strongly regular.

2. SYMMETRY PROBLEM: THE ORBITS

Symmetry is a more essential attribute of graph structure. It is there treated on its **B**-meaning [6], that expressed as existence *elements of the same kind*, specifically elements on the *equal positions* in structure.

2.1. Orbits: equal positions in the structure

Wherein appear the equal positions of structural elements? It appears in the *equal remain structures* that obtained after removal the equal elements. By different elements obtain different remain structures.